The New Geometry

Liber de geometria nova et compendiosa by Blessed Raymond Lull

God, You are the supreme truth, wisdom and love. With your virtue and in your honor, we now begin this new and concise Geometry.

As it is lovable to discover things quickly, let us now briefly investigate the secrets and natural truths of sensible and imaginable measurements, and let us carry out this investigation following the process of the General Art. In this art, we want to use common, easy to understand terms, so that those who do not know the terms of ancient Geometry can understand this science, and thus we deal with plain numbers without entering into algorithmic formulas.

The divisions of this science

This science divides into two books.

The first book has three parts:

1. The first part deals with the squaring of the circle and the triangulature of the square.
2. The second part is about extending the lines of the circle, the square and the triangle.
3. The third part is about multiplying figures.

The second book has three parts:

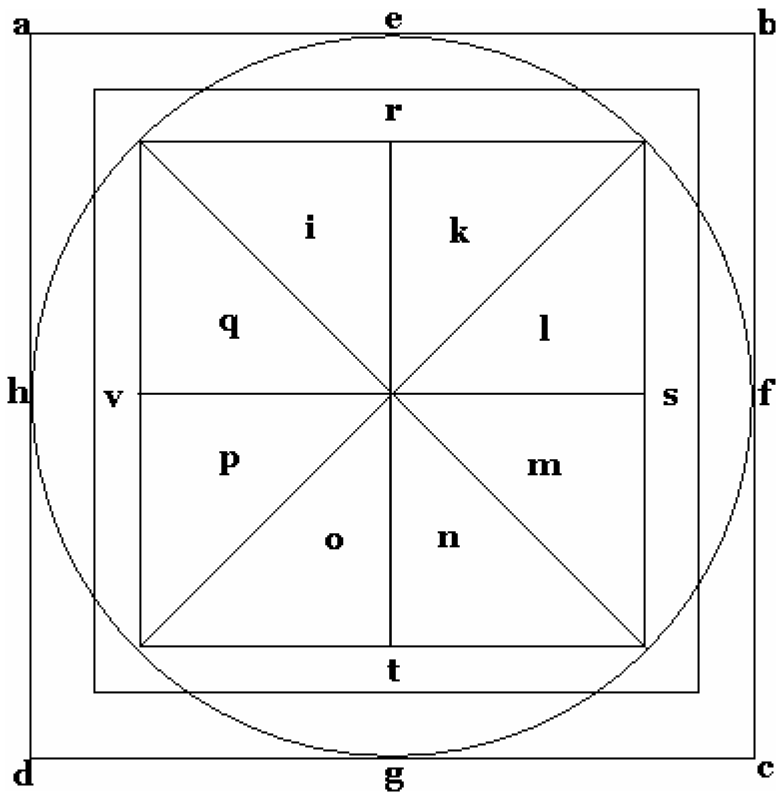
1. In the first part, we describe the usefulness of this science.
2. The second is about the principles of Geometry and the conclusions drawn from its principles.
3. The third part deals with questions and solutions of some doubtful issues of Geometry.

Book 1 - Squaring the Circle & Triangulating the Square.
Book 1, Part 1

This part divides into three parts: in the first, we investigate the squaring of the circle and its triangulature as well as the triangulature of the square, so let us begin with the first part.

Fig. 1 - The Master Figure

This part includes a figure called the master figure, and we call it the master figure because we will use it to investigate the truths of the other figures. It is composed of three squares and one circle, as shown.



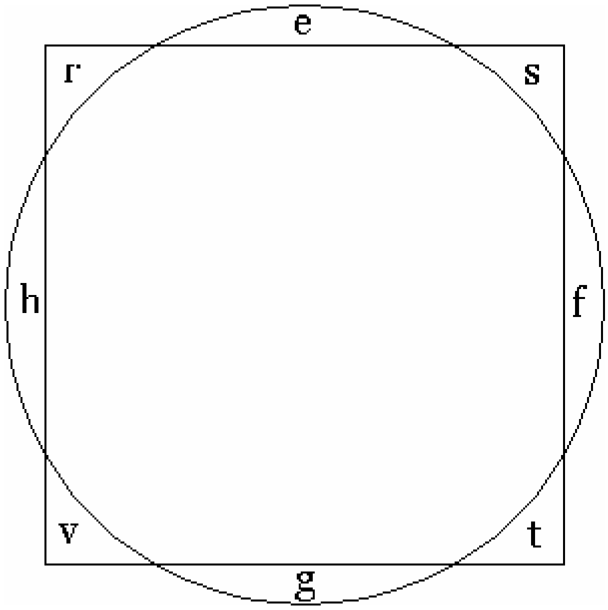
The minor square divides into eight houses or cameras equal in capacity, as shown, and it is contained in a circle, contained in turn by the major square signified by a. b. c. d. The circle stands in the middle between the major and minor squares and accordingly, it is signified by e. f. g. h. In addition, the circle has the value of the square signified by i. k. l. m. n. o. p. q., plus the value of e. f. g. h.

In the middle between the biggest and smallest squares, stands a square signified by r. s. t. v. This is the square we are investigating as the one in which the circle is squared, inasmuch as the circle and the square are visibly equivalent in content and capacity; but we want to prove this with the art in the following way.

If the circle between the biggest and smallest squares stands equally between both, or in other words, as close to the one as to the other as does square r. s. t. v., then the circle must have 12 houses, and the square must have 12 equal houses as well. Given that the third square stands in the middle between the two squares as does the circle, the third square in the middle has the same value as the circle, and the circle is squared in it.

To investigate further the squaring of the circle, we now say that the number halfway between 8 and 16 units must have 12 units. The circle is in the middle between a square worth 16 units and a square worth 8 units, where all units are equal to each other, therefore the circle must be divided into 12 units to match the sum of the units contained in the lesser square, plus its own units signified by e. f. g. h. Therefore, it is worth its own e. f. g. h. plus the value of i. k. l. m. n. o. p. q. Consequently, the circle is worth the 4 lines, or sides enclosing i. k. l. m. n. o. p. q. inasmuch as these 4 lines are circumferential. In addition, it is worth one of the four lines that make it worth more than the minor square, and this fifth line is the medium whereby the circle stands at an equal distance between a.b.c.d. and i.k.l.m.n.o.p.q.

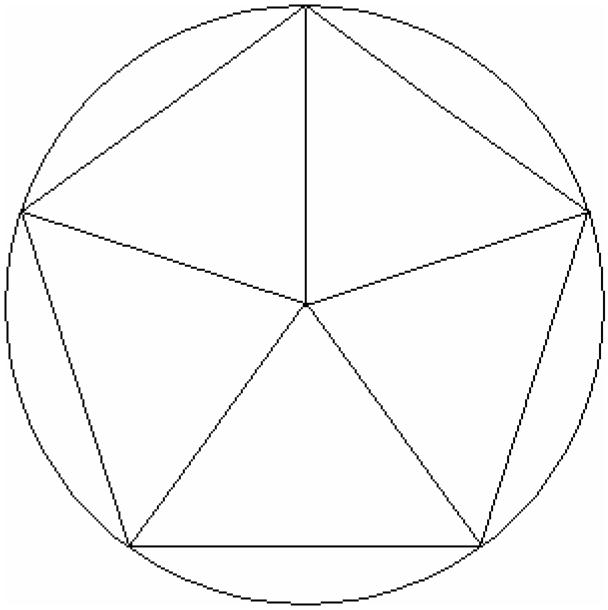
Fig. 2



This circle is worth the 4 lines which are the sides of the minor square, plus another one equal to any one of the 4, as is shown next in Figure 2. With a compass, draw a straight line whose 5 units are worth e.f.g.h.i. k.l.m.n.o.p.q., and then divide it into 4 equal units or lines. Then draw a square worth r.s.t.v., which we can sense, and let us put one foot of the compass in the center of this square, and with the other foot, let us draw a circle of the same value as the circle in the master figure. Now we have the circle in which the square is circled and the square in which the circle is squared, both equivalent in capacity.

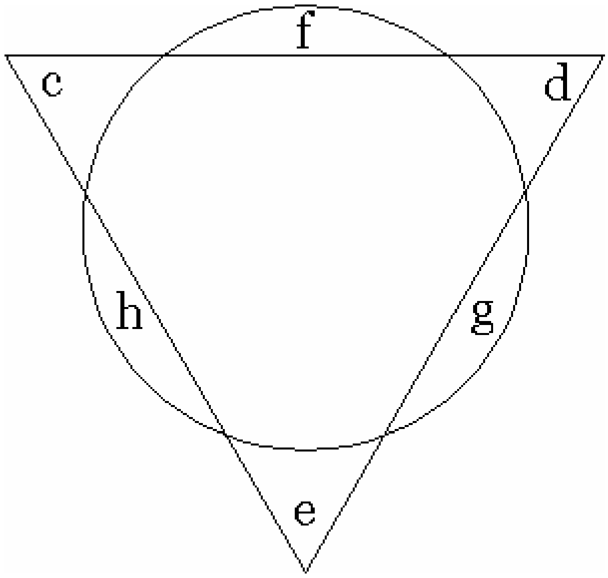
Given that e.f.g.h. and r.s.t.v. are equivalent, we have proved that the circle is squared by a 5th line added to the 4 lines of the minor square, which gives 5 lines that we make into one straight line x.y. That neither more nor less than a fifth unit must be added to the 4 units of the minor square, is shown by square r.s.t.u. in the master figure, where it stands between squares a.b.c.d. and i.k.l.m.n.o.p.q. as does circle e.f.g.h. Indeed, if the long line x.y. were not simply made of 5 constituent lines, and if it were worth more or less, it could not be made into a square standing halfway between 8 and 16, as do e.f.g.h. and r.s.t.v.

Fig. 3



Further, with this pentagonal figure, we want to prove that line x.y. divided into four equal parts makes a tetragon that squares the circle. We prove it as follows: from the 5 circumferential lines contained in the pentagon plus a sixth line, let us make one straight line worth as much as line x.y., not more and not less. We can clearly sense this when measuring it with a compass. Therefore, we must know that the square made from line x.y. has the same containing capacity as circle e.f.g.h., for if it did not amount to this, but to something more or less, it would not be equivalent to the straight line made from the pentagon and from the sixth line. The pentagon served to prove that its 5 lines plus one line are worth line x.y. The same method of proof applies to the six-angled figure, and to figures with seven or eight angles. This can be proved to the senses with a compass by adding a line to each successive figure, as we exemplified with the pentagonal figure in long geometry, and we call this 'long geometry' because in it we multiply the principles of other sciences.

Fig. 4 – The Triangulature of the Circle



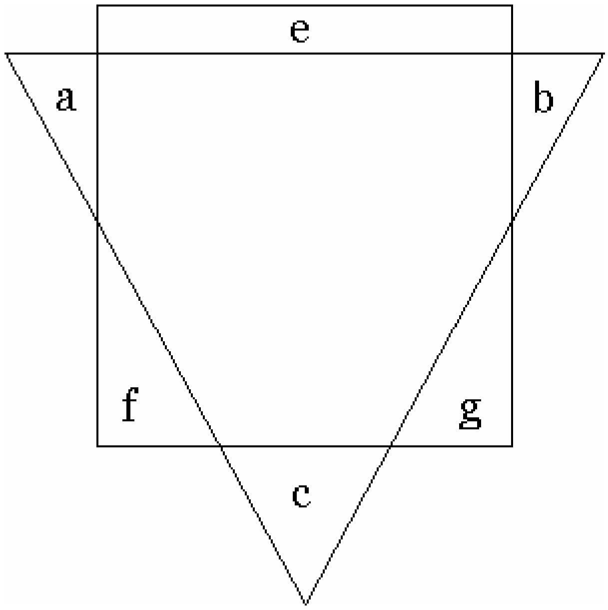
To investigate the triangulature of the circle, where the circle contains as much as the triangle and the triangle as much as the circle, we must first consider that a right angle belonging to a square has more containing capacity than an acute angle belonging to a triangle. This is why the triangle has longer lines and measures than the square, given that a triangle cannot contain as much surface as a square when the length of lines is the same for each figure.

Therefore, since the triangle needs longer lines than does the square, we should consider that the capacity of the square is halfway between the capacity of the circle and triangle, just as circle e.f.g.h. stands halfway between the major and minor squares. Now let us look for the measurements we want in order to draw a triangle that triangulates the circle. Here is how we take these measurements.

From the minor square in the master figure, take one of the diametrical lines that participate in two angles touching the circle. Now from 4 diametrical lines equal to this diametrical line, make one straight line a.b., divide it into three equal lines, and from all three make an equilateral triangle. Put one foot of the compass in the center of the triangle, and with the other foot, draw a circle equal to circle e.f.g.h. This triangle triangulates the circle, because the triangle's c.d.e. match the circle's f.g.h., as the eye can see.

We saw that the capacity of the square is halfway between the capacities of the circle and triangle. Now square r.s.t.v. has four lines as we said, and the triangle is made of four diametrical lines. As the square's capacity is halfway between those of the circle and the triangle, line x.y. must be worth a half line more than the line of circle r.s.t.v. and a half line less than line a.b. which encloses the triangle in which the circle is equally triangulated. Therefore, the square's capacity stands halfway between the triangle and the circle by one line composed of two equal halves. The square's line is greater than the mentally extended circular line by one-half, and smaller than line a.b. by the other half. Hence, we proved that these are the right and necessary measurements for triangulating the circle; indeed, if there were more or less of them, the square's capacity could not be halfway between the capacities of the circle and the triangle.

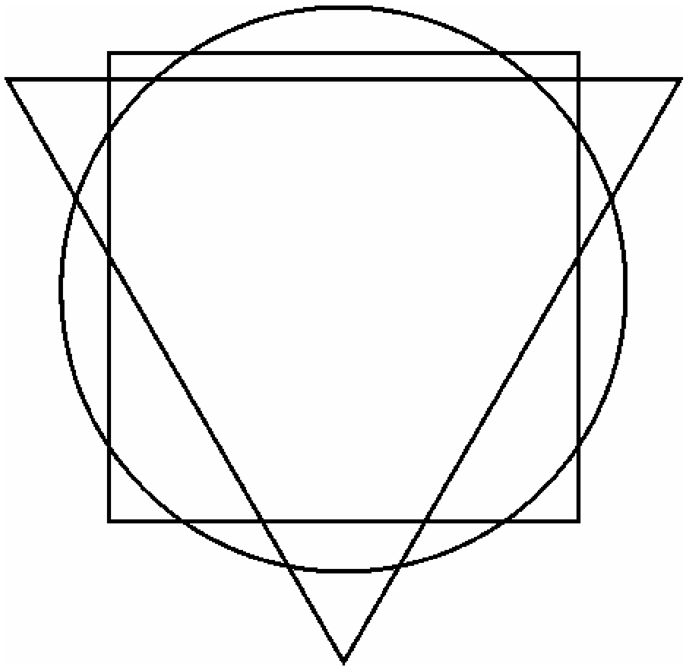
Figure 5 – The Triangulature of the Square



We proved the squaring and the triangulature of the circle; now, following these two proofs, we want to prove the triangulature of the square, where the selfsame circle has the same value as one square, and the square has the same value as the circle. Now if the same relation of value exists between the circle and the triangle, then the square and the triangle must necessarily be equivalent. If they were not equivalent, they could not match the selfsame circle, therefore one triangle and one square are equivalent, and we can sense this, inasmuch as a., b. and c. signify that they are worth the surface covered by e.f.g.

To put the square and the triangle together, we must first draw a triangle, then find its center, and make this center coincide with that of the square. Thus, the triangle and the square are equivalent in capacity, and they have the selfsame center, as they mutually contain each other.

Fig. 6 - The Plenary Figure



We call this the plenary figure because it is composed of a circle, a square and a triangle, all equivalent in capacity. We also call it the plenary figure, because the circle, the square and the triangle all share one center. Now these figures are general to all figures, just as all the elements are made of simple elements. Likewise, all compound figures, be they natural or artificial, descend and derive from the circular, square and triangular figures. We see this, for instance, in the human form, or in the shape of a shield in which some parts signify the circle, others the square, and others the triangle.

To draw this plenary figure, first draw the triangle, then the square, and then the circle, so that all 3 figures have the same center and each one is centered on the others.

Book 1, Part 2

Extending the lines of the circle, square & triangle

In this part, we propose to investigate the extension of circular lines, and we make this imaginary extension because a circular line, be it natural or artificially drawn with a compass, cannot be extended across a surface.

Now let us see how this investigation proceeds. In the first part, where we proved the quadrature and triangulature of the circle, we said that the capacity of the square is halfway between the capacity of the circle and that of the triangle. Thus we proved that line x.y., which is worth 4 units of the circle in the master figure, is worth half a unit more than the circular line of the circle, and that line a.b. is worth one half unit more than x.y.

Thus, we see that if we could extend the line of the circle in the plenary figure could, it would be worth only three and a half units of line x.y. in which the square extends. Additionally, line a.b. would be worth one unit of the square more than the circle. Thus, in the mind's eye, the extended circular line is half a unit shorter than the line of the square, and one unit shorter than the line of the triangle, while all the figures are equal in capacity.

Following what we said about the extended line of the whole circle that is worth eight half-units of the square, we can discuss the parts of the lunules in the master figure. Let us take the circular line e. which has the same value as the straight line ik, and as much more as the fourth part of seven and a half units, or as a quarter unit of the straight line i.k.

We showed how circular lines extend and how their extended value compares with that of straight lines. Now, following the example we gave with the master figure, we can give examples with figures of five, six or more angles, following the natural measurements of the capacity of the circle, square and triangle, and we just discussed this capacity here, in Part two of this book.

At this point, we can consider how science originates in imaginary quantities drawn from quantities that the senses perceive, as when we make an imaginary measurement of a circular line by straightening it, and mentally measuring it with the straight lines of the square and the triangle. These considerations expand the imagination's virtue as it feeds and verifies it with imaginary species; in this way, one attains natural secrets inherent to the composition of figures, and we will give examples of this in the third part.

Book 1, Part 3 - multiplying figures

This part is about figures derived from the prime ones, here we show how some figures are derived from others, and how one figure can be measured with another; and we show how some parts are equivalent to others in one and the same figure.

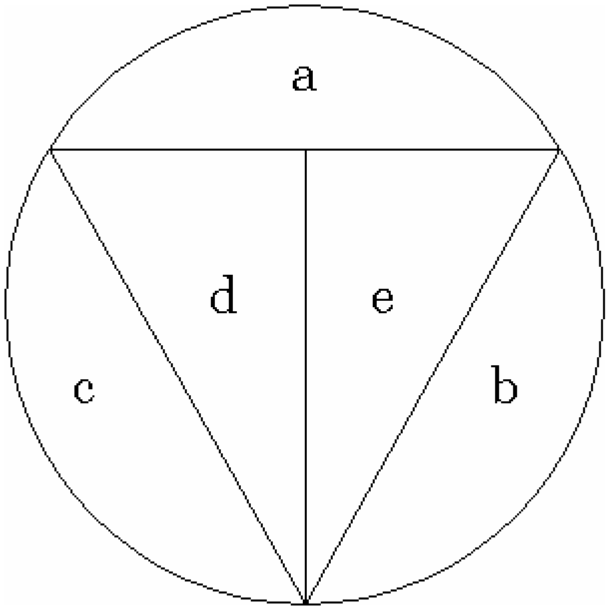


Figure 7

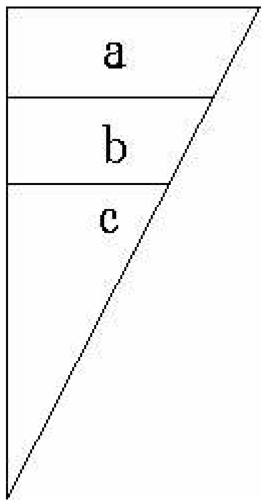
Let us divide this figure, made of an equilateral triangle enclosed in a circle, into 5 equal parts, which are equivalent to one another as the eye can see, and let us take a half of the triangle from this figure.

Fig. 8 - the half-triangle

This figure of the half-triangle is divided into 3 equal parts a, b and c as shown, and each house is equivalent to every other house, with regard to the figure's capacity and surface.

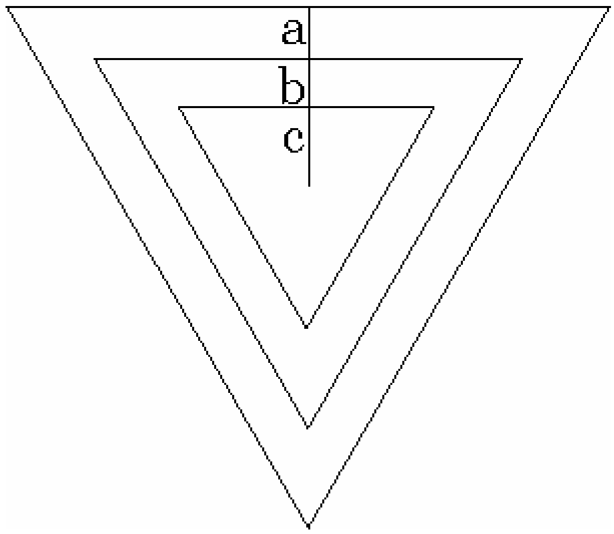
This figure is an instrument used to form equal parts in other figures, and we want to prove the equivalence of a., b. and c. as follows.

The first part about the master figure shows that circle e.f.g.h. is halfway between the major and minor squares. For this reason, we say that house b. in this figure is halfway between houses a. and c., inamuch as in this figure, we are dealing with the 16 equal houses of the master figure, of which c. has 9, b. has 4 and a. 3.



The reason why c. should have 9 houses is that its angles are more acute than the angles of b. And given that c., by the nature of its angles, has the nature of the angles of i.k.l.m.n.o.p.q. in the master figure, it has 8 units per se and because of the acuteness of its angles, it acquires one unit from b., whose angles are more similar to a circle than those of c. And as we subtract one unit from a.b. and give it to c., we subtract one unit from a. and give it to b., because house a. is more extended in length than house b. which is more similar to a square than house a. Therefore, c. is in the 9th degree of 16 units, b. in the 13th and a. in the 16th, where c. has 9 units, b. has 4 and a. has 3.

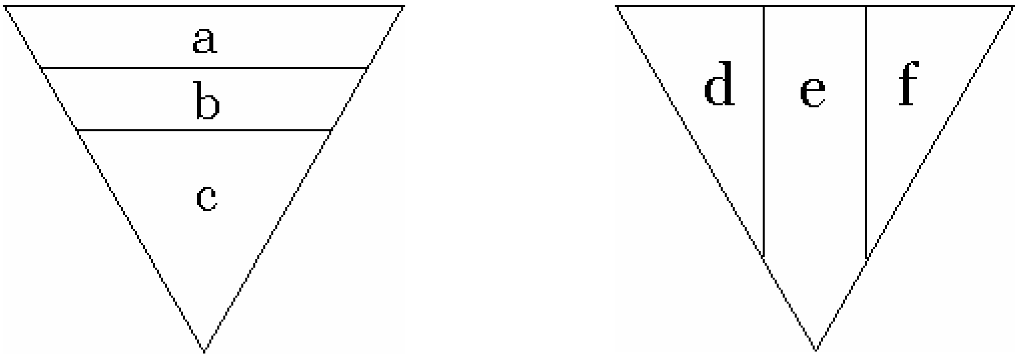
Fig. 9 - three triangles



This figure is composed of three triangles, any one has the same containing capacity as any other, and the half- triangle serves to measure it, since its vertical line and the radius of this figure are equal in quantity, and triangle a. is in the 16th degree, triangle b. in the 13th and triangle c. in the 9th.

This provides a doctrine and method for using the half-triangle to place triangles within one another, where the contained triangle is equivalent to the containing triangle, and so this figure is useful for reciprocal measurements, where one measure is equivalent to another in essence, nature and virtue.

Fig. 10-11 – triangles



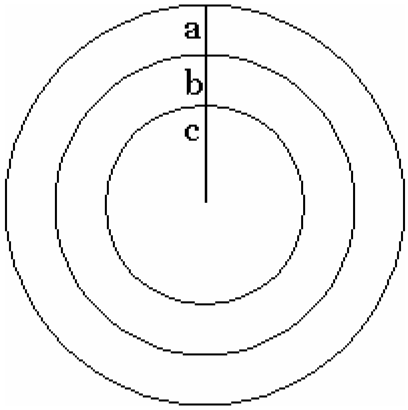
These two triangles divide into equal parts so that the house of any letter is equivalent to the house of any other letter. Here, triangle a.b.c. arises from duplicating the measure of the half-triangle to make a whole triangle, with house a. standing in the 16th degree, house b. in the 13th and house c. in the ninth.

Another triangle, d.e.f. has a similar measurement, as follows. From the 16 equal houses of the master figure, make eight double units and from them all, make a straight line, and then divide it into 5 equal parts.

From the 5, give 2 to d. and 1 to e. as it has acute parts only in the lower angles. Here, e. has four instances of right angles, two internal and two external. However, d.f. only have only two, so that we must give 4 of the 5 measures to d.f. and 1 to e.

These figures are useful inasmuch as they provide a doctrine for measuring longitudinal and transversal parts that are equivalent in the same subject, with one instrument, called the half-triangle.

Fig. 12 - three circles

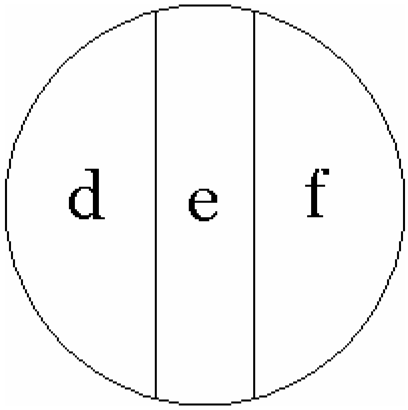


This figure of three circles provides a doctrine for placing circles within one another, where any circle is equivalent in containing capacity to any other, regardless of the fact that one is located inside another.

To measure this figure's equal parts, we follow the method we used in the preceding figure of 3 triangles. It is useful to know the measurements of this figure, as it teaches the way to place one circle inside another so that the contained circle is equivalent in quantity, essence, form and matter to the one containing it.

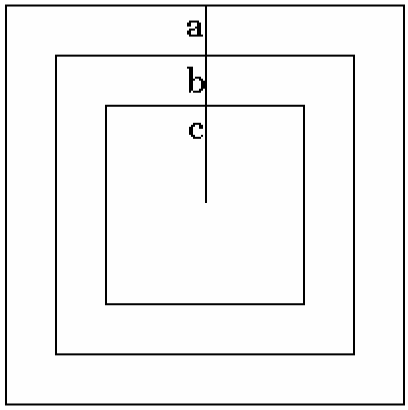
Moreover, the senses can tell that house a. is equivalent to house b., and house b. is equivalent to house c. Further, this figure is useful for equating the circle essentially and naturally with the triangle, given that a. in this figure is equivalent to a. in the figure of three triangles, and the same with b. and c.

Fig. 13 - a circle



This figure, d.e.f., serves in this art to provide a doctrine for dividing the circle into 3 essentially and substantially equal parts. This division follows the method of the master figure and the method of the half- triangle. We draw one diametrical line straight across the surface and divide it into 8 double parts worth the 16 houses of the master figure of this art, and give 3 of these 8 parts to d., 2 to e. and 3 to f. This is because e. occupies a longer lengthwise space than d.f.

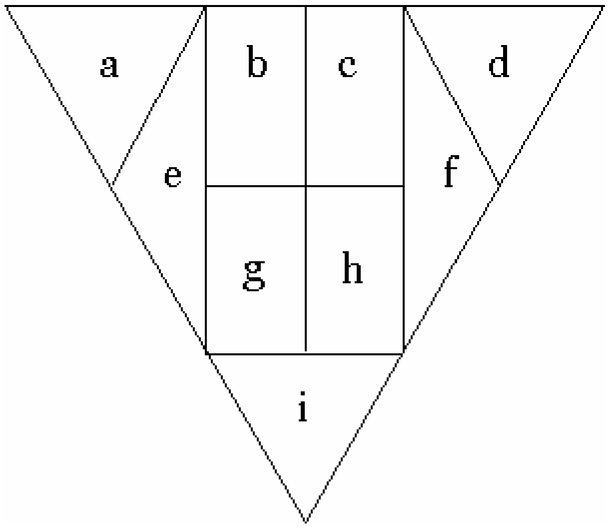
Fig. 14 - three squares



This figure provides a doctrine for placing a square within another square so that the contained square has the same substantial value as the one containing it. In measuring this figure, we follow the method we used in the figure of three triangles and the figure of three circles so that the straight diametrical line, which extends from the upper surface to the center of the square, is divided in the same way as the half-triangle figure.

Thus, a. in this figure of squares is equivalent to a. in the figure of triangles and a. in the figure of circles, and the same with b.c. This figure, like the two said figures, is useful to natural philosophers and geometers, because it shows that triangular substances are equivalent to circular substances, which allows us to determine the quantitative mixtures of figures and values, and this knowledge is a great contribution to philosophy.

Fig. 15 - triangles & squares



This figure is made of 9 cameras, all equivalent in containing capacity, as shown by a.b.c.d.e.f.g.h.i., and it is included in this art to provide a doctrine for acquiring an art and a method for equating cameras of diverse figures under one and the same figure. For instance, a.b.e. in this figure are equivalent, and consist of a diversity of situations and angles, because a. is made of acute angles, b. of right angles and e. of 2 acute angles and 1 obtuse angle.

By the mixture and situation of various cameras, geometers can know the composition of the said angles, and physiognomists can understand human forms, and astronomers can know the figures of stars and the situation of the influences they transmit to things below, and natural scientists can know the situations that the simple elements have in compounds. Further, this figure is good for knowing other figures, for instance: a. in this figure is a fifth part of b.c.g.h.

Fig. 16-17-18 - circles & triangles

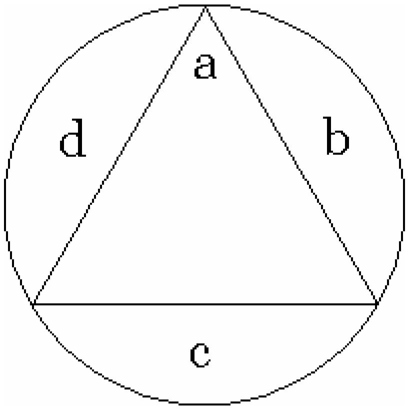


Fig 16

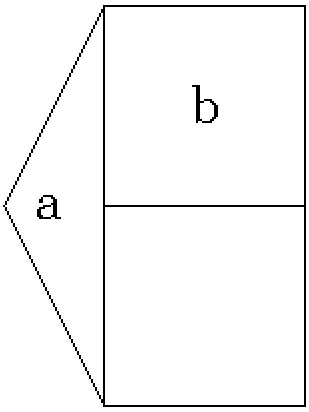


Fig 17

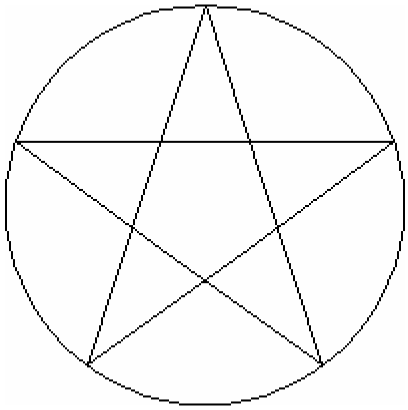
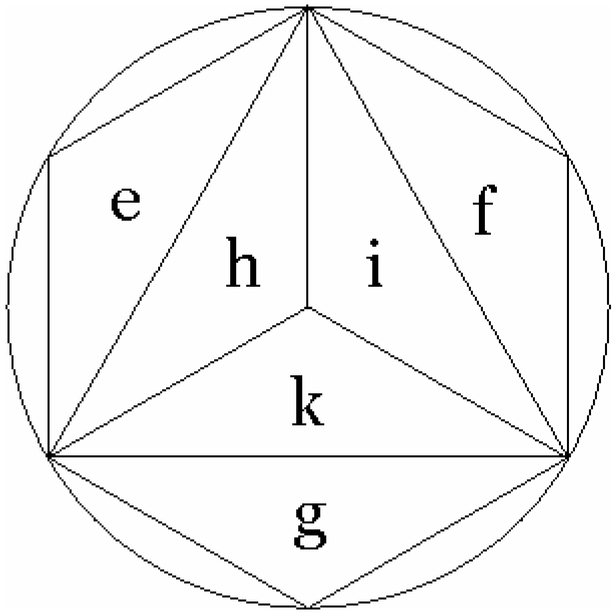


Fig 18

This figure of the circle and triangle is included in this science as an instrument for measuring other figures, as by the 3 lines of a triangle and 1 line equal to any one of the 3, the circle can be squared as we said in the Major Geometry. Likewise, with triangle a. we measure the lines of the figure of Solomon as follows. With a compass, make one straight line worth the 3 lines of triangle a. plus one equal to any one of the 3. Then, make one straight line with the 5 straight lines in Solomon's figure, worth one unit each, and we find that the 5 lines of Solomon's figure are worth 1½ units more than the 4 lines of a. Given that the line of Solomon's figure is worth 1½ units more than the line of a., which is worth four, let us now measure the circular line in Solomon's figure, supposing we use triangle a. to extend it. The straight line made of four units is worth line x.y., and it is worth a ½ unit more than the line of the circle in the master figure. Now the 5 units of Solomon's figure are worth 1½ units more than line x.y., and the 5 units of Solomon's figure are worth 2 units more than the circular line of Solomon's figure, and this can be experimentally verified with a compass. The above figure of the circle and triangle signified by a. is also good for investigating the values of the lunules of circles, namely in the pentagon, hexagon, heptagon and octagon, and we provide experimental arguments for it in the next figure.

Fig. 19 – a figure for measuring lunules

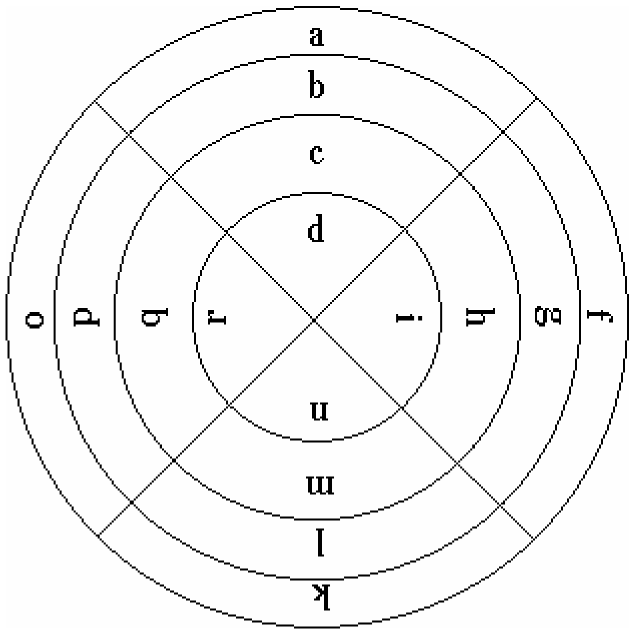


It is clear that the triangle in figure 16 potentially holds the division of the triangle shown in the present figure, divided into three equal parts signified by h.i.k., which are equivalent to houses e.f.g. as the eye can see.

Therefore, the triangle in the 16th figure with a. in it is worth 3 parts of the circle, and the 3 lunules b.c.d. of the same figure are worth more than a. inasmuch as the present figure has 6 lunules which the other potentially holds. Consequently, this figure e.f.g.h.i.k. shows us the potential divisions in the other figure. Moreover, the potential divisions in the other figure can bring the divisions of figure e.f.g.h.i.k from potentiality into act.

This figure, discovered by using the 16th figure, provides a doctrine for knowing the extremities of circles. The 6 said lunules make up the 7th part of the circle, and we will prove that this is so as follows. Now house e. is one part of the heptagon, and house f. is another, and so on with g.h.i.k. Here, every part is equal to every other, while together they add up to the number 6, hence the number must necessarily increase to 7 on account of the lunules, so that the entire circle can be divided successively into equal parts. We dealt with this in a similar way in the Major Geometry, in the 5th circle designated by the letter a. Therefore, the six lunules make up the seventh part of the whole circle. Thus, there is a succession of numbers from the sixth unit to the seventh, which could not be if the six lunules did not add up to the 7th part of the circle. If they added up to 6, then house k. and the 6 lunules would both add up to the same number, which is impossible. Moreover, if they added up to an 8th part, the succession from six to seven would be destroyed, but the natural numbering of equal parts cannot sustain any such destruction. Further, if the 6 lunules added up to one half-unit above k., the unity of numeric succession would be destroyed, inasmuch as there could be no integral increase of numbers by whole units like the one that occurs in the number made of e.f.g.h.i.k.

Fig. 20 - the elemental spheres



This figure is included in this science to provide a doctrine enabling one to imagine the spheres of the 4 elements and the spheres of the 7 planets in accordance with their quantitative, substantial and essential natural values; and this according to their quantitative, essential and specific natural values.

Here, we follow the order of the half- triangle and the figure of three circles that we already drew, where a contained circle is equivalent to a containing one. In this figure, a. signifies the sphere of fire, b. the sphere of air, c. the sphere of water, and d. the sphere of earth, and likewise with the other letters according to the regions they occupy, and the figure signifies the entire body that fills all the space within the lunar sphere.

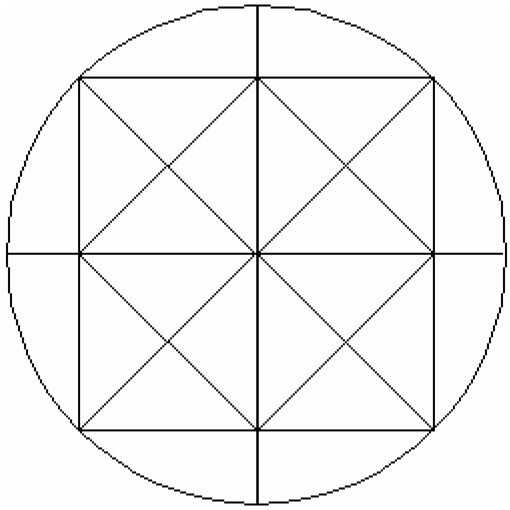
Four diametric lines divide the figure into four parts, so that the figure represents the world’s division and extension into 4 regions.

Because fire has more virtue than air, it must have more form and less matter than air, and likewise with air as compared to water, and the same with water and earth, and due to this, d. must be a thick and concave body, and likewise with a.b. Now natural order requires that c. be thicker than b. and b.

than a., so that d. can be contained by c. and c. by b. and b. by a., and here we can realize that elemental form has more extensive capacity than matter, occupies more space than matter, and has a subtler nature.

As we imagine the layout of this figure, we can also imagine the situation of the planetary spheres, where the lunar sphere is thicker than the sphere of Mercury, and the sphere of Mercury thicker than the sphere of Venus, and so on with the rest of the planets. Now this must be true, to ensure natural mutual correspondance in the order in which planets and elements exist, so that things here below can better receive the influences from above. With this figure, one can imagine the situation of vapors and winds when they form acute and wide angles as they ascend and descend. For instance: angle d. is acute as compared to angle c., and angle c. is acute as compared to angle b. and likewise with a. This is why vapors expand and dilate as they ascend, and when they descend, they contract and condense to take up less space. Therefore, the gross and heavy vapors that descend below, create winds made of subtler and lighter vapors moved by the upper vapors whose imprint they bear, as they have an appetite to descend to things below and occupy places that contain vapors from which winds materially arise that cannot move up and down without moving transversely.

Fig. 21 - the elemental degrees



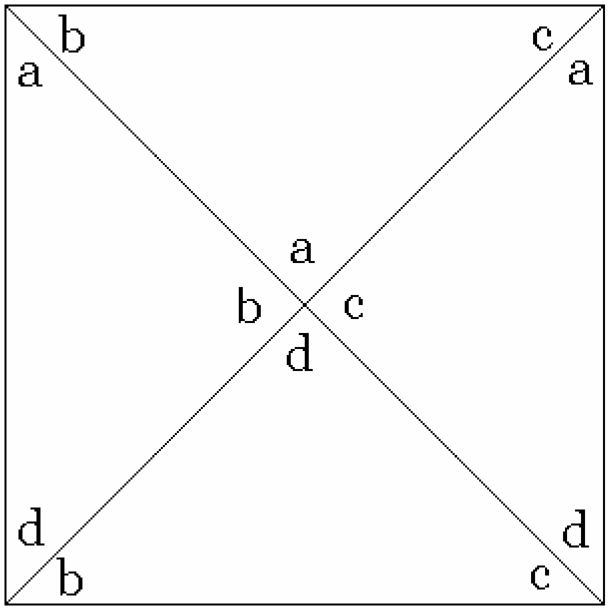
This figure has 24 equal houses as shown, and its 24 houses signify the 24 hours of the natural day, which are equal here although they are not equal in artificial days and nights.

This figure is included in this science to signify the disposition of the degrees in which elements exist in elemented things, as for instance in a peppercorn, or some other plant in which some element is in the fourth degree; and we now give an example of this as follows.

In a peppercorn, fire is in the fourth degree of heat, earth is in the third degree of dryness, air in the second degree of moisture and water in the first degree of cold. For this reason, we give one of the 24 houses to fire in the peppercorn, we give another house to earth, another one to air and another one to water. We do this to represent their simplicity, and after this distribution, we make a further distribution of the 24 houses as follows.

In pepper, we give four houses to fire because fire in pepper is in the 4th degree of heat. We give 3 to earth because earth in pepper is in the 3d degree of dryness; and we give 2 houses to air because it is in the 2nd degree of moisture, and we give 1 other house to water because it is in the 1st degree of cold. Then, we do the same with the 10 remaining houses. Thus, fire in pepper has 9 houses, earth has 7, air 5 and water 3. By this rationale, pepper assumes its conditions according to the division we described, and fire rules in pepper, and earth rules after fire. This figure, clarified in this way, signifies the degrees and composition of elements in elemented things, it is a most useful figure for physicians who know how to graduate medicines with it, and it is most delightful to natural scientists who want to know how the elements enter into composition.

Fig. 22 - elemental mixture



In this figure, a. stands for fire, b. for air, c. for earth and d. for water. We attribute the triangle with a. near the center to fire, the triangle with b. near the center to air, the triangle with d. near the center to water, and likewise with earth.

Now fire heats air, and receives dryness from earth. Therefore, in the triangle of fire, a.b. are in a concordant aspect, as well as a.c., but b. and c. are in a contrary aspect. Three measurements of action and passion arise here, each of which is of a dual nature, for fire instills its own quality of heat into air and it also instills its dryness that it receives from earth.

Therefore, the aspect between a. and b. involves 2 actions and 2 passions. One involves the concordance between fire and air, whence generation arises in elemented and generated substances like animals, plants and metals; and through the contrariety between a. and b. due to dryness and moisture, ensues the corruption of old substances and the disposition to generate new substances.

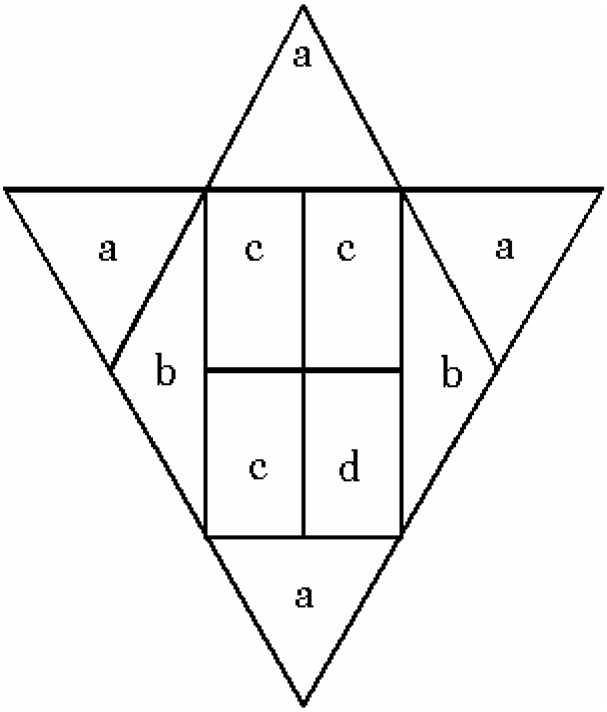
What we said about a.b. also applies to a.c., and the aspect between b. and c. is entirely contrary because it is made of dryness and moisture. Therefore line b.c. consists entirely of contrary qualities and measures.

What we said about triangle a. also applies to triangles b.c.d. For instance, b. receives heat from a. and gives its moisture to water, and so it heats water inasmuch as air entering into water carries heated moisture with it. Thus, as air enters into water, water receives three qualities, namely moisture, heat and dryness. Likewise, all the elements enter into one another through their proper and appropriated qualities, while no element ever abandons its own quality, which it cannot relinquish because it is its own subject.

Following what we have shown with a.b.c.d., you can, in your imagination, know and measure the quantities of action, passion, influx and reflux of the elements in compounds through generation and corruption. You can also know how the simple elements enter into one another through composition. Here, each simple element retains its own essential nature even as it exists in composition throughout the said mixture. In a coin made of gold, silver, copper and tin, all 4 metals are compounded, melded and merged into each other as parts in parts and as parts of the whole, where the whole is the coin, while each metal retains its own essence and natural being as the subject of its own quality and natural virtue.

This figure is useful for physicians to know, as well as for natural scientists and geometers, inasmuch as physicians can procure health following the conditions proper to the elements and the way they enter into mixture; and natural scientists can fathom the secrets of nature. In addition, geometers can use it to imagine the behavior of simple and compound points, and then go on to the lines, situations and figures displayed by substance.

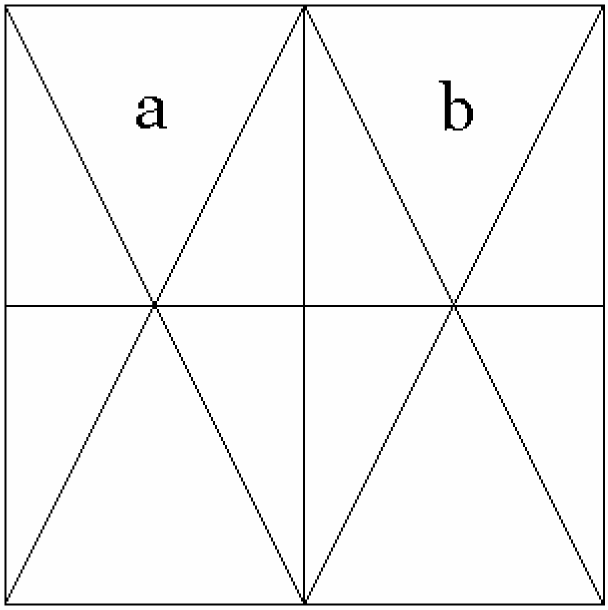
Fig 23 – the 4th degree of heat



This figure signifies a plant in the 4th degree of heat, such as pepper, garlic or scammony. Here, a. signifies four measures of heat, c. 3 measures of dryness, and b. 2 measures of moisture, so that this figure signifies equally measured degrees in elemented things. This signification enables one to measure the equal and natural measures that some elements have more than others in elemented substances in which some elements are in higher degrees than others in essence, nature and substance. Here, the essence of fire and its nature, substance and virtue exist in greater quantity inasmuch as there are 4 a's; and the essence of earth, its substance and virtue exist in greater quantity inasmuch as there are 3 c's; and air exists in greater quantity with its 2 b's, and water with its 1 d.

And all these quantities are mixed together through natural generation, corruption and composition, in the same way as in a furnace, are mixed and melded together 4 ounces of gold, 3 of copper, 2 of tin and 1 of silver. With this figure, natural philosophers can know that when the simple elements enter into composition, they change in situation and shape while the essence, being and nature of each remain actual. We know by experience that an alchemist can take a melded mixture of gold, silver and copper and separate the metals out; and one can also do this with water and wine, i.e. separate them out.

Fig 24 - intensive & extended measurements

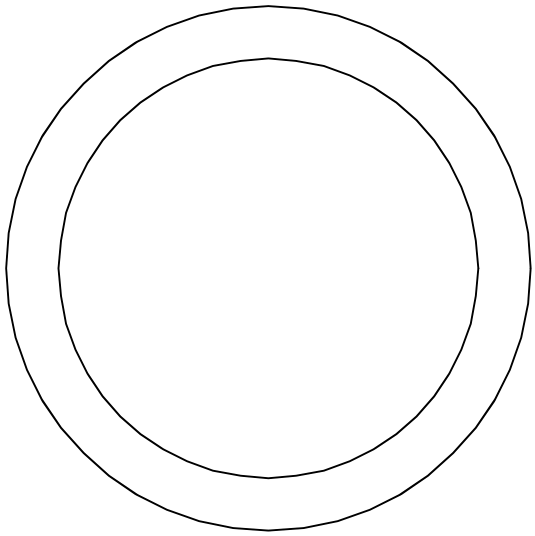


This figure signifies that the intensity of elements in concordance or contrariety increases as they approach one another in form and virtue. Here, in the two centers in the square figure signified by a.b. there are six angles where the elements are closer to each other than in the middle center of the square that has four right angles.

This shows that when elements are in highly intense mutual participation, they participate because each element's matter keeps it away from the others, given that substance occupies more space with its matter than with its form.

This figure is good for examining the intensive or extended mixture in the urine of patients and women. In addition, it is good for geometers who can imagine how the closeness between the major lines is greater in some places than in others, as the eye can see in a.b.

Fig 25 - the white circle



In this science, this figure gives a doctrine for considering how the white circle potentially holds points, straight and oblique lines, right, acute and obtuse angles, from which derive the figures existing in the species of elemented corporeal substances. You can consider these things in accordance with what we said about the figures of the degrees of elements, and other figures dealing with natural operations.

In the figure of elemental mixture, we said that a. and b. stand for fire related to air in the concordance of heat and moisture, and this concordance causes a straight line and a right angle quadrangular in nature. Because fire instills its dryness into air, its influx causes an acute angle triangular in nature, given that air and fire oppose each other through moisture and dryness.

Now as air agrees with fire through heat, it declines from the right angle and makes an acute angle inasmuch as it restricts itself so that earth, the proper subject of dryness, cannot enter into it. Now as air seeks help and sustenance, it inclines toward water that helps it against fire, and here it makes an obtuse angle which is confused because it is composed of a right angle and an acute one. The remaining elements make angles in the same way as air, and generate circular, straight and oblique lines as their constituent points bring them from potentiality into act.

This is how circular figures arise, where every element enters successively into every other element. With this figure, geometers can imagine the principles at the source of the oblique derivation of angles, and of oblique, straight and circular lines with their constituent points. The white circle potentially holds all these artificial measurements because the natural properties within its substance are available to natural agents, so the artist can draw to the surface of the circle the likenesses of lines, points and angles contained within it.

At this point, there is much subject matter for philosophy, because here, many natural secrets reveal themselves to the human intellect that desires to consider subtly how likenesses pass from potentiality into act, part by part, and to know the natural inner operations of elemented things.

Fig. 26-27 - measuring the moon

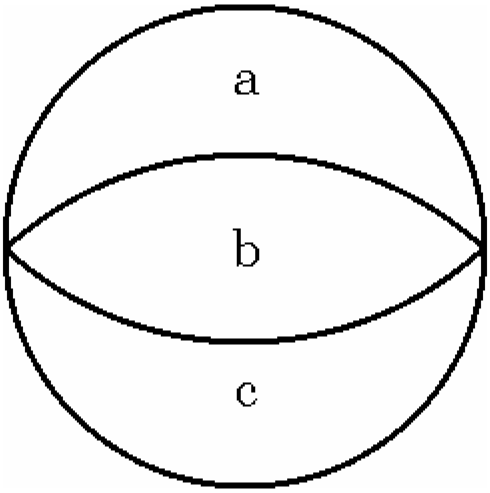


Fig 26

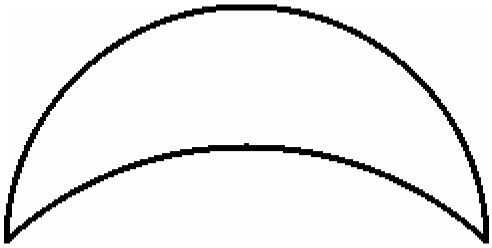
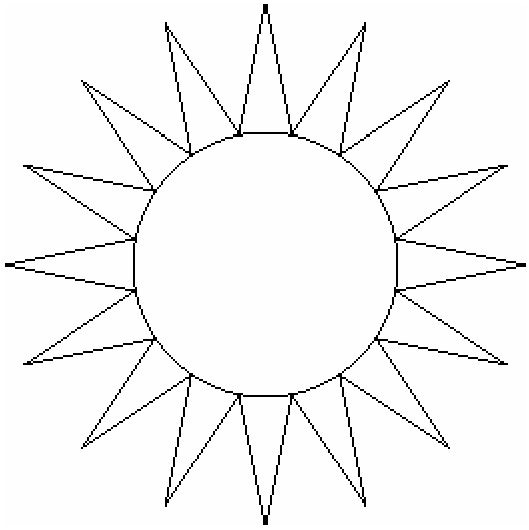


Fig 27

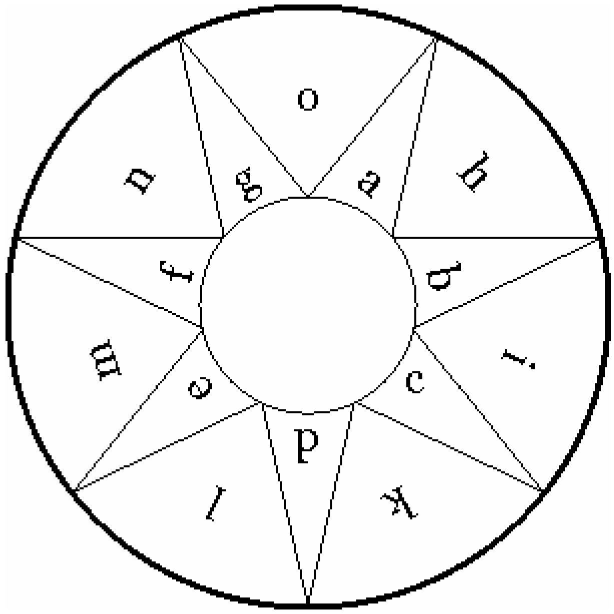
This figure is included in this science so you can use it to visualize the full moon divided into 3 equal parts, as can be seen in a.b.c., and with this visualization you can imagine that when the moon appears as in figure a., one third of the moon is lit up by the sun. This figure of the moon is included in this science to determine when the sun lights up one third of the moon, reckoned by the moon's size at the time when it appears in the figure that is most naturally its own. Just as an angle has more capacity through perpendicular straight lines than through oblique ones, so likewise, the rightest and most proper figure of the moon is the one it takes on when one third of it is illuminated, as the eye can see. Therefore, when the moon reaches the point where it appears in its rightest and most proper figure, its virtue must be more intense than at any point or time in which it is in a less proper figure.

Fig. 28 - a star



This figure is included in this science for reckoning the proportional disposition of influences instilled by the stars in things below, as these influences follow the order between causes and their effects. There are 4 elements, and each element has 4 specific degrees in elemented things. Fire has the fourth degree of heat in pepper, the third degree of heat in cinnamon, the second in fennel and the first in anise. Therefore, the star must have four rays with which it instills its virtue into fire, in the same the way as fire has its degrees in plants. With one ray, the star satisfies the fourth degree of heat, and the third degree with another ray, and so on in sequence. The star must have four other rays with which it satisfies air, and likewise with water and earth. Hence, the star must have 16 rays for instilling its virtue more strongly in some elements than in others, in an order determined by the degrees held by elements in elemented things, as discussed in the figure of the degrees of the elements. Therefore, it is useful for natural philosophers and astronomers to know the conditions and circumstances of this figure.

Fig. 29 - the planets

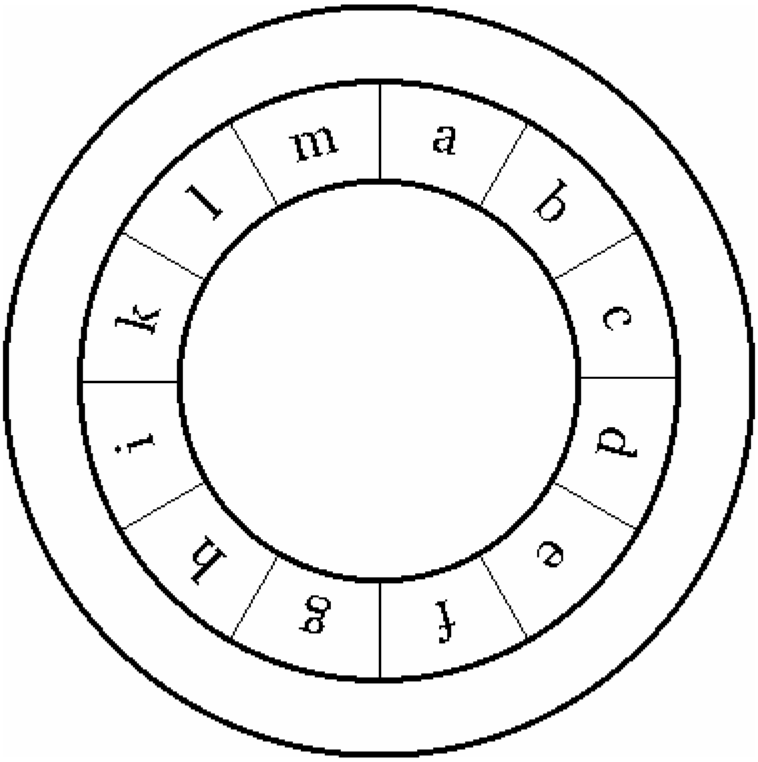


This figure of the planets, in this art, provides a doctrine for metaphorically measuring the influences of planets on things here below, and we want to discuss this influence briefly as follows: a. in this figure stands for Saturn, b. for Jupiter, c. for Mars, d. for the Sun, e. for Venus, f. for Mercury and g. for the Moon.

We say that h. and i.k.l.m.n.o. are regions into which the planets instill their virtues. Now, a. has a cold and dry earthy complexion, b. has an airy complexion, and when a. instills its influence straight into h. as when a. is at midheaven, it instills more virtue than when it instills it laterally. We have experienced this with a burning mirror by which the Sun transmits more of its heat through acute angles than through obtuse ones. Therefore, as a. directly instills its virtue in its opposite through an acute angle, it sends its influence along a straight line. When its opposite is oblique, for instance, if h. is closer to b. than to a., then the influence of a. decreases proportionally because b. occupies h. and a. does not influence as strongly through a transversal line than through a perpendicular one.

The things said about a.b.h. apply likewise to the remaining lines, according to their conditions. If region h. has an earthy complexion, a. can instill more of its virtue into it than if h. has a complexion opposite to that of earth. Now a. can instill more dryness than cold, because dryness is earth's proper quality, whereas cold is appropriated to earth by water. With all these things in mind, this figure of the planets is useful for imagining their influence and for mentally measuring major and minor, straight and oblique influences.

Fig. 30 - the 12 signs



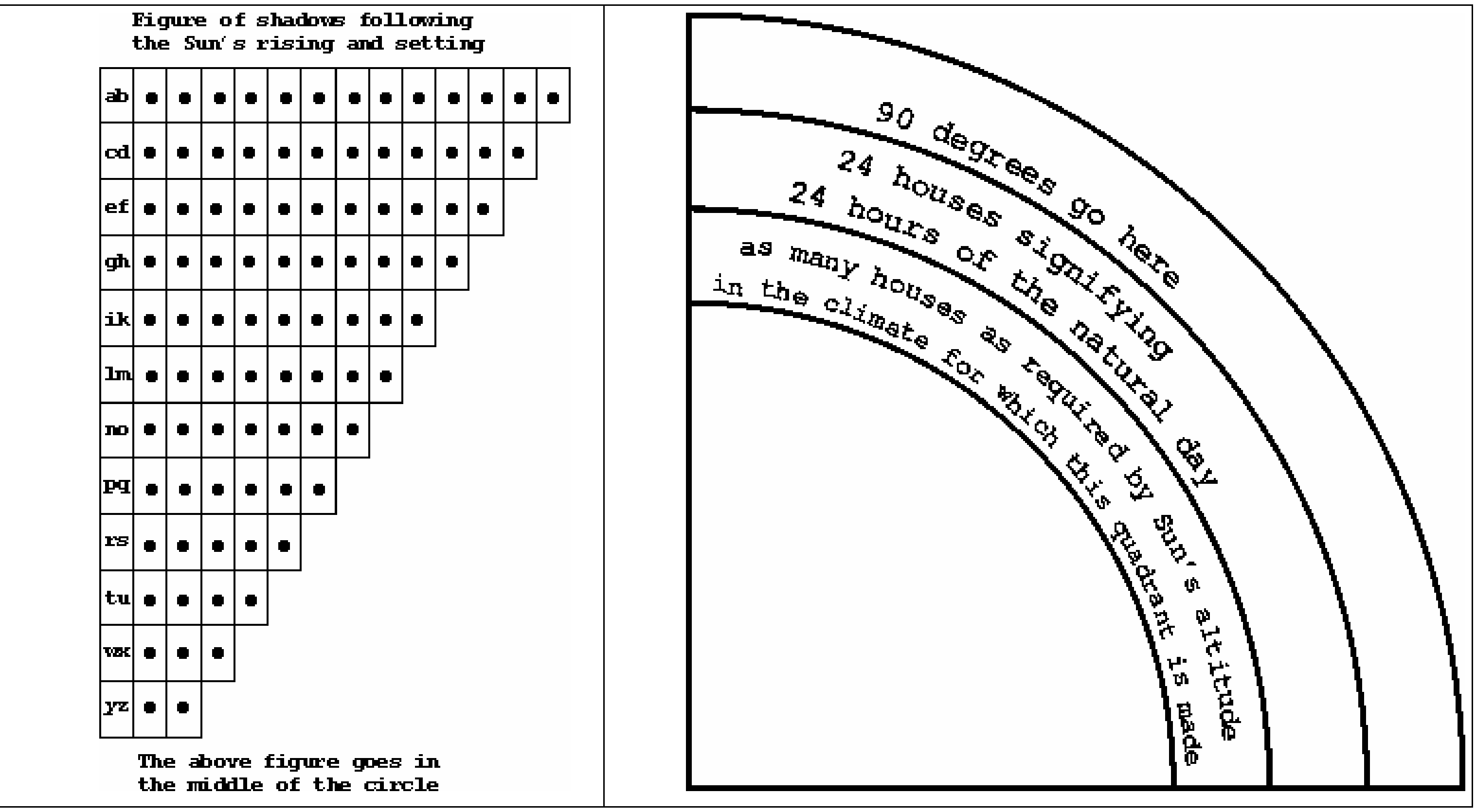
This figure of the 12 signs of heaven is included in this science for you to visualise the distances of heaven and the planets, and to visualise constellations. However, we will speak briefly about this science here because we already dealt with it at length in the New Astronomy we wrote.

Now, a. stands for the complexion of fire, b. for earth, c. for air and d. for water. The complexion of Aries is a., Taurus has b., Gemini c., Cancer d., Leo a., Virgo b., Libra c., Scorpio d., Sagittarius a., Capricorn b., Aquarius c. and Pisces d.

For instance, when Saturn is in the house of Aries, the constellation is hot and dry with dry and cold, and the two instances of dryness in this constellation make earth its ruler, over an unfortunate opposition between hot and cold qualities, which is confused and cannot last very long. If hot and dry Mars enters into this constellation, then the constellation has three instances of dryness, two of heat and one of cold, which is fortunate for earth, whereas water suffers enduring misfortune. If moist and warm Gemini enters into this constellation, then the constellation has three instances of dryness. However, the one instance of cold and one of moisture are afflicted, whereas heat and dryness are fortunate, and earth most of all, since fire is dry because of earth, so that moisture in this constellation is more afflicted than cold, because dryness in this constellation is more fortunate than fire. Following the example we gave with the said signs and planets, you can understand other constellations.

In the master figure, we saw that the circle is halfway between the major and minor squares, meaning that there are four units between 8 and 12, and 4 units between 12 and 16. Due to the orderly concordance between the quintessence and the four general substances of the world, namely the 4 elements, there must be order in heaven as there is order in the combined descent of the circle, triangle and square. Hence, there is an equal space of four equal units between the Moon and the outer surface of the sphere of fire, and from Saturn to heaven, with the planets in the middle between heaven and the sphere of fire. Since the natural order between a cause and its effect truly require this disposition, it is good to know this figure of the 12 signs and to visualize it following the process we described.

Fig. 31 – a quadrant for telling time by day



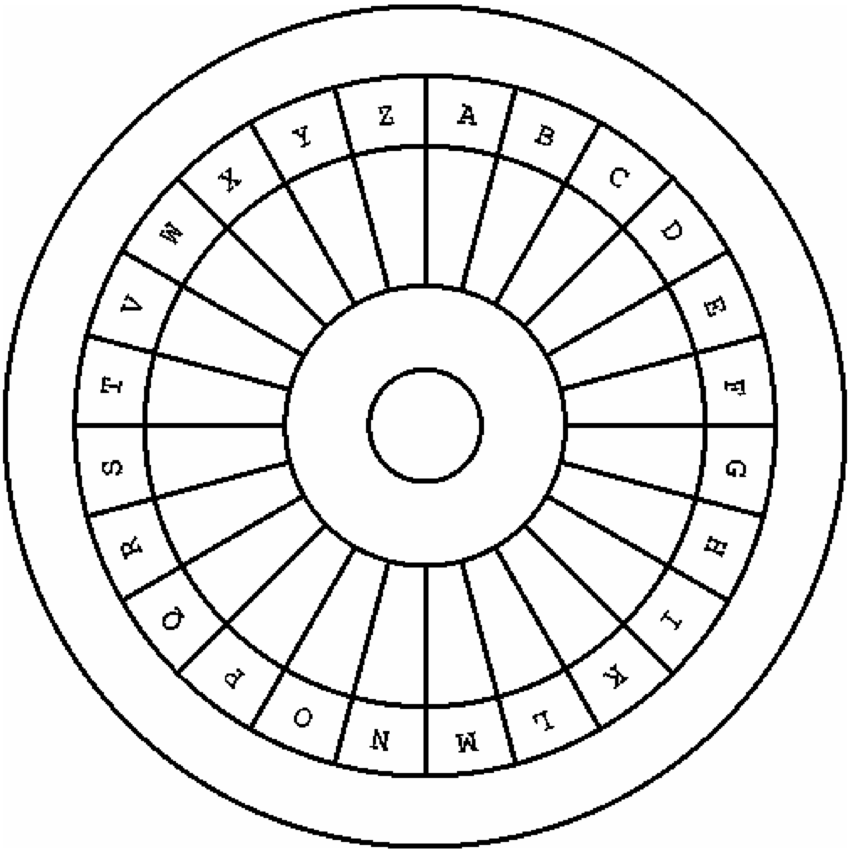
In the quadrant, there are three circles: one divides into 90 degrees, another into 24 parts, each part is called a half-hour. Another circle contains 24 lines signifying the hours, for instance, a. signifies the first half hour of daylight as well as the final half-hour at sunset; b. stands for the second half-hour of the morning and the second last half-hour of daylight. The same applies to the other hours, each in its own way. Now as a. is a rising half-hour and b. is another rising half-hour, a.b. make up one hour in the morning, which is the first in the day. Then, as they descend, a.b. make up another hour which is the last hour of daylight; c.d. make up the second ascending hour of daylight, and the second descending evening hour; e.f. make up the third ascending morning hour and the third descending evening hour; and so on in sequence from the first letter to the last.Because the sun casts a longer shadow near sunrise and sunset than in the third hour, or at noon, or in the ninth hour, and as it casts longer shadows in the ninth hour than at noon, we must investigate and determine how many of the 90 degrees belong to each daylight hour. Now the first hour has more degrees than the second hour because the shadow is longer in the first hour than in the second. Likewise, the second hour has more degrees than the third, because the sun casts a longer shadow in the second hour than in the third, and the eye can see this at sunrise and sunset when the sun casts gigantic shadows of people, and at noon when people's shadows are just the size of people. In addition, the same applies to the third hour, the fourth, the fifth, and so on.

The degrees divide the arc according to the artificial day. A day takes 90 degrees in the equatorial region where an artificial day in the month of June has 24 hours. Now the division into hours follows the equatorial day.

> Give 6½ degrees up the edge of the quadrant to the first hour signified by a.b., and to the final hour of daylight, at sunset, give another 6½ degrees, so that a.b. have 13 out of 90 degrees shared between 2 hours.
> Give 6 ascending degrees to the second hour of daylight and to the second last hour of daylight give another 6 degrees, give 11 degrees in ascent and descent to the hours signified by e.f.
> Give 10 degrees in ascent and descent to hour g.h.
> Give 9 degrees in ascent and descent to hour i.k.
> Give 8 degrees in ascent and descent to hour l.m.
> Give 7 degrees in ascent and descent to hour n.o.
> Give 6 in ascent and descent to hour p.q.
> Give 5 degrees in ascent and descent to hour r.s.
> Give 4 degrees in ascent and descent to hour t.u.
> Give 3 degrees in ascent and descent to hour w.x.
> Give 2 degrees in ascent and descent to hour y.z.

This division includes all 90 degrees in the 24 hours of the day, no other division can do this, or else there would have to be more or less than 24. Now you can know this by experience, as I did.

Fig. 32 – an instrument for telling time at night



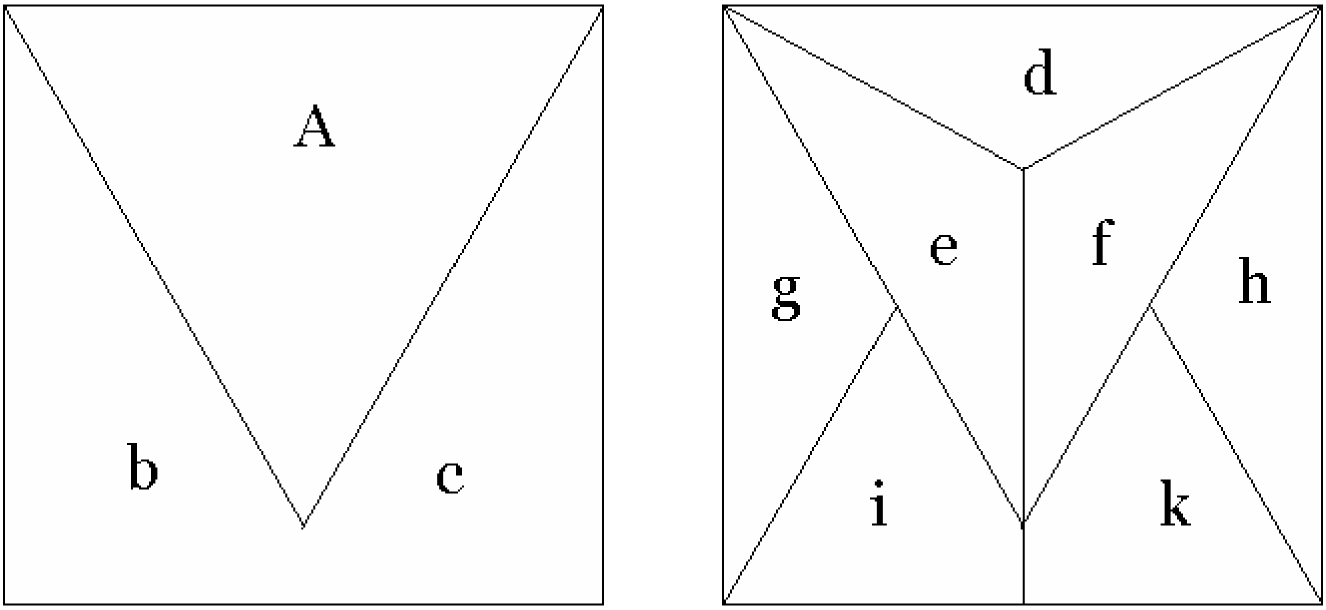
This figure is included in this science for telling time by night, which you can do in the following way. Make one large copper circle containing a small one, and divide it into 24 equal parts, called hours, designated by a.b.c.d.e.f.g.h.i.k.l.m.n.o.p.q.r.s.t.v.w.x.y.z. The houses between the large and small circles are perforated. Both circles are connected by 12 straight lines, and the small circle in the middle should have a hole in the center through which you can look at the Pole Star while closing one eye and holding the circle toward the Pole Star. Hold the circle half a palm away from the eye and at half a palm's distance measured obliquely from a. to the forehead, and one palm from g. to the beard, to get a good, equal view of the circumference of the sky and of the Pole Star through the apertures in the houses.

After sunset, as the Pole Star begins to appear, and you see the two stars that sailors call the Two Brothers revolving around the Pole Star, and as the Greater Brother appears in some house or other, this is where you begin to compute the first hour of night. Then watch the Greater Brother moving successively from one house to the next, while at the same time you watch the Pole Star through the hole in the small circle. If night has 9 hours and day has 15, and if the star called the Brother appears at sunset in house a., then by dawn, when the stars begin to fade, it will have moved to camera i. Likewise, if it appears in house b., it will have moved to house k. However, if the night has 10 hours, and the star we call the Brother appears in house a. in the first hour of night, then by the last hour of night it will have moved to house k., and so with the others in turn, until the night that is 15 hours long.

Suppose that the night is 9, or 10, or more hours long, and the Brother appears in a. in the first hour of night. Now someone who has slept, or stayed up at night wants to know how long he slept or stayed up, he can look at the house where the Brother has moved. If he slept for 3 hours, it has moved from a. to c., and if he slept for 4 hours, it has moved from a. to d., and so forth. If he wants to know how many hours there are until daybreak, and if the night has 10 hours and he slept for 3 hours, he can reckon that there are 7 hours until dawn, and so forth.

This instrument is useful for knowing how long you have slept, or stayed up at night, and how long you can continue to sleep, or stay up until sunrise; it is useful for night travelers by land or sea. This knowledge is useful, for instance, to someone who awakes too early and believes that he has overslept, or conversely.

Fig. 33-34 - triangular & square measurements



These figures are included in this science for measuring lines with other lines, like the 3 lines of A. measured with a compass and then drawn out into a single straight line, and likewise with the whole square, and the square is worth one measure more than the triangle.

Thus, a doctrine is provided in this science whereby, in the triangular and quadrangular figures, you measure the lines of the triangle with a compass, and draw them out into a single straight line, and you do likewise with the square containing the equilateral triangle which is the largest that this square can contain. Then the eye can see that the square is worth four of seven equal measures, and the triangle is worth three as shown in the second figure d.e.f.g.h.i.k. that divides into 7 equal parts as the eye can see. With this figure, one can know that a square is worth one-fourth part more than an equilateral triangle made of lines equal to those of the square containing it. Likewise, a circle containing an equilateral triangle touching its circumference is worth one-fourth part more than the triangle. We dealt with similar matters in the other Geometry in the part on Circle a.

Fig. 35 - a fortress

This is a good figure to measure, and we want to use it to provide a doctrine enabling one to find the right measures for artificial structures according to their use and proportion, like the pentagonal figure, which is more appropriate to the form of a fortress than any other figure. Now in the pentagon, one measure is the means of four measures. Likewise, the fortress walls must have equal measures in all four squares, and the tower's width must be of one measure equal to any of the 5, and its height must be of 5 measures equal to the 5 measures of the walls, so that they are worth 5 measures vertically and 5 horizontally in all the squares. Such a proportioned shape cannot exist in a square figure because the tower would be too short, and if the figure were hexagonal, then it would have the same number as the walls and would not be proportioned in the middle of the squares. If the fortress were configured in a heptagonal figure, the tower could not stand in the middle of the 4 squares in a numerically natural way as it does with the pentagonal figure.

Fig 36 - a tower

FIGURE OF A TOWER

The figure of a tower, in its proportions, requires the figure of the square more than that of the pentagon or any other figure. This is due to the nature of the four quadrants it has in its square figure, and it requires four vertical measures equal to the four quadrants measured transversely, which give it its angles. A tower cannot have such proportions between its quadrants and its height, except in the square figure, given that no figure but the square figure is made of four; so that no tower is as strong in war as a triangular one. Now the lines of an acute angle are closer together than are those of a right angle, and in a tower with three angles, the angles are closer to each other than in a tower with four angles. Therefore, angles that are closer to one another can help each other better than angles that are farther apart; and the same applies to the three vertical measures that can help three angles better than four vertical measures can help four angles of a tower.

Fig. 37 - a church or palace with a tower, or a hall with no tower

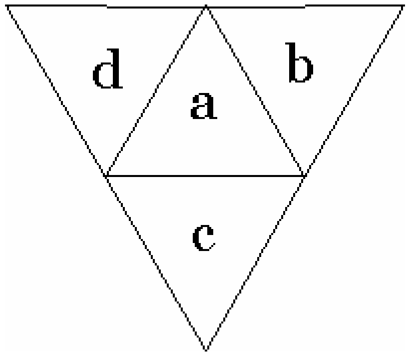
The figure of a church or a palace with a tower, or the figure of a hall without a tower derives from the eight-angled figure more properly than from any other figure. Here the belfry or tower is eight measures high and two wide, which make up a quarter of eight. The church's body is four measures high and eight measures long so that it is as long as the tower is high and four measures in breadth so that it is immediately longer than it is wide, and it is four measures high so that the length matches the height. This proportionate measurement cannot exist in any other figure that could be as well suited to the purpose of a church, or palace with a tower, or a hall without a tower, as the figure with eight angles, as shown in this figure, drawn according to the said measurements.

Fig. 38 - a bedroom

A bedroom is a place for rest and sleep, and no figure is as well suited to its purpose as the heptagon. We see this in this figure, which is seven measures long, two measures wide, and three measures high including the roof. Now the reason why the best-suited figure of all is a heptagonal one is that a bedroom cannot concord with any other figure in view of the purpose for which it provides shade. If its shape were quadrangular, its form would not be conducive to rest. A person reclining in the room in which the master is reclining would be too close to him. Moreover, if it were pentangular, two measures wide and five measures long, it would be too wide. If it were one measure wide and five long, it would be too narrow and would not have the right form. The same applies to the six-angle figure as two measures of width would make it too wide and one measure would make it too narrow, and likewise with the figure of eight angles.

We have spoken of the method of measurement suited to buildings according to the figures from which they must derive in view of their function and use. From the things we said about the said figures, you could derive a doctrine for suitably configuring the construction of houses, portals and public places, and this doctrine is useful for those who love beautiful and well-proportioned buildings.

Fig. 39 - three equilateral triangles



This figure serves in this science to provide a doctrine for measuring lines contained by other lines, as in the smaller triangle whose three lines are only worth a half of the three lines of the bigger, general triangle, and likewise with b.c.d.

You can know this by experience: make one straight line with a compass out of the three lines of the major triangle. Then make another straight line out of the three lines of a, and you will see that the line of the greater triangle is worth twice the line of the smaller one. The same will happen if you draw 3 triangles in a. like those in the major triangle, and so on successively until the smallest triangle that can possibly be made.

From this, you can derive a doctrine for multiplying triangles that contain more than the contained ones, and the same considerations apply to squares drawn within each other. We are done with the first part where we showed how to quadrangulate and triangulate a circle, and how to tringulate a square. We made 39 figures to provide a doctrine enabling one to obtain an art and a method for measuring and imagining mathematical quantities by using quantities that the senses percieve, and for measuring one figure with another. The doctrine we gave with these 39 figures can serve to study other, peregrine figures.

Book 2

The second book has three parts:

1. In the first part, we describe the usefulness of this science.
2. The second is about the principles of Geometry and the conclusions drawn from its principles.
3. The third part deals with questions, and solutions of some doubtful issues of Geometry.

Book 2, Part 1 - The Usefulness of this Science

In the first part of this book, we describe the usefulness of this science as follows. It is clear to the wise that science begins in the sensitive and imaginative powers where the human intellect receives species that are likenesses of sense objects. Then it makes these species intelligible in its essence, and due to this intelligibility, the intellect attains the natural secrets of corporeal substances. In addition, as the intellect cannot do this without help from the imagination, this science also nurtures the imagination in imagining imaginary mensurations based on sense data.

Therefore, this science is good for strengthening the imagination's act of imagining and the intellect's act of understanding, and likewise with memory, for when the imagination is more virtuously disposed in imagining, the memory is more virtuously disposed in retaining the species transmitted to it by the intellect and acquired through the imagination.

Through the master figure, this science gives a doctrine for penetrating the quadrangulature and triangulature of the circle, and it gives a doctrine for the other figures as well. Thus, the imagination develops its virtue inasmuch as it attains new species in the equality of circular and straight lines and their surfaces, following the doctrine given in the first book. In addition, by multiplying species attained by the imagination through diverse investigative methods, the intellect and memory acquire much material that enables them to reach a greater understanding of corporeal substances and accidents.

For instance, you extend the line of the circle to consider it as a straight line. Likewise, in the minor square, you extend a fifth line measured in the master figure by the intellect with the imagination. Take this fifth line along with the four other lines to produce the middle square of the master figure. We proved this in the second figure where the circle is squared, and likewise with the sixth mathematical line of the pentagon in third figure, and so with the other figures used in this science to imagine mensurations that are not, and cannot be sensed. Through this, the imagination has a loftier and more virtuous act than with lines and figures that the senses perceive.

For instance, the fifth line of the square potentially exists in the circle of the master figure, and is more useful to imagine than the four lines of the minor square in the master figure. This kind of imagining raises the imagination aloft and helps the intellect and memory to better attain imaginable objects that you cannot really imagine as such. You can understand and love such objects spiritually, like God, angels and general or abstract principles like general goodness and other such things, following the comments given in the first book on each particular figure and the things deduced from them.

Moreover, this science is useful, as we said, and as it is understandable per se, it is very dear to those who desire to know it and who desire to have an intellect that understands well and a memory that remembers well.

Book 2, Part 2 - Principles of Geometry & Conclusions drawn from them

This part about the principles of Geometry provides a doctrine for drawing conclusions in Geometry, and for solving questions. We want do deal with these principles under 10 rubrics, with 10 principles under each rubric. And given that mensuration is the subject of Geometry, and measurements are made with lines, and lines consist of points, we will first deal with points, by first defining the first principle, and then by deriving principles from its conditions, and we will follow the same process with the remaining principles.

Points

Principles of Points

1. A point is an entity that is part of a line.
2. Without a common point, no line can participate with another line.
3. Every central point is a common one.
4. A point without parts is indivisible.
5. Any point that is the substance of an angle is common and divisible.
6. An indivisible point cannot be sensed or have parts.
7. A point is always wider in a right angle than in an acute angle.
8. Only mathematical points are simple.
9. Without a common point, no line existing within the ultimate surface of heaven can have an end.
10. Without points, motion is impossible.

Corollaries to the Principles of Points

1. We said that a point is an entity that is part of a line. Hence, it follows that all sensible lines are compound, given that all things that have parts are compound.

2. We said that without a common point, no line could participate with another line. Therefore, given that surfaces are made of participating contiguous lines, every surface must have parts.

3. We said that every central point is a common one. Hence, it follows that every common point is a body, or has a corporeal nature, and has length, breadth and depth if it is a body, because without a body and the three natural dimensions of physical growth, no corporeal point can be common.

4. We said that a point without parts is indivisible. Hence, it follows that such a point cannot be physical, and that it must be a primordial, simple point, the kind of point that the prime general constituent parts of nature have. On these points, nature builds sensible physical substances from prime principles by using compound points, such as natural substantial physical goodness, and likewise with greatness, duration, power, instinct, appetite and other such primordial simple principles from which are derived the compound principles common to lines, surfaces and depth.

5. We said that any point that is the substance of an angle is common and divisible. This leads to the conclusion that there is some corporeal point that divides down to a primordial indivisible corporeal point like the one we described in the 4th paragraph. For instance, you cannot divide a point such as natural goodness any further into essences or species; for it is a general essence that never gives up its specificity.

6. We said that an indivisible point cannot be sensed or have parts. Hence, it follows that prime indivisible points can be neither seen, nor heard, nor touched, like natural goodness, greatness, duration and other things that cannot be divided into parts, because if they could be divided, they would be

neither simple nor general, but would be bodies as such. If this were so, they would be at the same time simple and compound, general and not general, which is impossible.

7. We said that a point is always wider in a right angle than in an acute angle. Hence, we conclude that the point is a genus that includes many species, like wide points and acute points, which are distinct due to distinct species of angles distinguished by the distinction between the triangle and the square, and this shows that species are real essences.

8. We said that only mathematical points are simple. Hence, it follows that abstract mathematical essences are simple, like one essence of fire, one essence of heat, or one potential form, and so with other things like these.

9. We said that without a common point, no line existing within the ultimate surface of heaven could have an end. Hence, it follows that a point that terminates a line must be a body, because if it were not a body, it would have no termination.

10. We said that without points, motion is impossible. Hence, it follows that motion always proceeds successively from some single point.

Lines

Principles of Lines

1. A line is the surface of a figure.

2. Lines consist materially of points, figures consist materially of lines.

3. Surfaces consist materially of contiguous lines.

4. A line cannot exist without fullness.

5. All lines consist of long points.

6. There is no line as continuous as a circular line.

7. All lines have neighboring lines.

8. There is no line that expresses natural appetite as well as a straight line.

9. As a middle line moves toward one extreme, it moves away from the other extreme.

10. Every natural square line stands in the middle between a circular line and a triangular line.

Corollaries to the Principles of Lines

1. We said that a line is the surface of a figure. Hence, it follows that lines bound all figures.

2. We said that lines consist materially of points. Hence, it follows that all lines are compound.

3. We said that surfaces consist materially of contiguous lines. Hence, we conclude that surface has matter.

4. We said that a line could not exist without fullness. Hence, it follows that fullness consists of full points in lines.

5. We said that all lines consist of long points. Hence, it follows that all surfaces are made of wide points.

6. We said that there is no line as continuous as a circular line. Hence, it follows that a circular line is fuller of points than any other line.

7. We said that all lines have neighboring lines. Hence, it follows that all lines terminate somewhere.

8. We said that there is no line that expresses natural appetite as well as a straight line. Hence, it follows that nature gives greater appetite through straight points than through oblique points.

9. We said that as a middle line moves toward one extreme, it moves away from the other extreme. Hence, we conclude that every line occupies a locus.

10. We said that every natural square line stands in the middle between a circular line and a triangular line. Hence, it follows that all the influence from a circular line that goes to a triangular line, passes through a square line.

Angles

Principles of Angles

1. Angles are full parts of squares and triangles.

2. A right angle has more capacity than an acute one.

3. All obtuse angles are confused.

4. Without angles, all bodies would be round.

5. Without angles, there can be no surface.

6. An angle cannot exist without a common point and without lines.

7. All right angles potentially hold acute angles, but not vice versa.

8. All acute angles are of a triangular nature.

9. There is no angle as general as that of the square.

10. At the center of every circle, more acute angles are disposed than right angles.

Corollaries to the Principles of Angles

1. We said that all angles are full parts of squares and triangles. Hence, it follows that all squares and triangles arise from angles.

2. We said that a right angle always has more containing capacity than an acute one. Hence, it follows that a square contains more with shorter lines than a triangle.

3. We said that all obtuse angles are confused. Hence, it follows that an obtuse angle belongs partly to the triangle, partly to the square and partly to the circle.

4. We said that without angles, all bodies would be round. Hence, it follows that as an angle is corrupted, its matter recedes to a round body.

5. We said that without angles, there can be no surface. Hence, it follows that the form of surface is made of angles.

6. We said that an angle could not exist without a common point and without lines. Hence, it follows that the point in every angle is composed of several simple points.

7. We said that all right angles potentially hold acute angles, but not vice versa. Hence, it follows that the right angle naturally precedes the acute angle, given that the things that do not convert with what is consequent to them are primordial.

8. We said that all acute angles are of a triangular nature. Hence, it follows that all acute points are of a triangular nature.

9. We said that no angle is as general as that of the square. Hence, it follows that the essence of the acute angle is of the essence of the right angle.

10. We said that at the center of every circle, more acute angles are disposed than right angles. Hence, it follows that the center of every circle is round.

Figures

Principles of Figures

1. A figure is a habit made of lines by the circle, the square and the triangle.

2. The figure of the circle is the mother of all figures.

3. There is no figure is as full as the circular one.

4. Because a pentagonal figure has broader angles than a square one, it is more similar to a circular figure than to a square one.

5. None of the general figures is as difficult for the imagination to visualise as the circle.

6. The circular, square and triangular figures are the only general figures.

7. No figure can exist without the circle.

8. Nothing represents the forms and differences of substances as well as a figure.

9. Figure is a likeness of form just as surface is a likeness of matter.

10. Without proportion among its parts, no figure can be beautiful.

Corollaries to the Principles of Figures

1. We said that figure is a habit constituted by the circle, the square and the triangle. Hence, it follows that the imagination cannot visualize a figure without the likenesses of the things that constitute the figure.

2. We said that the circular figure is the mother of all figures. Hence, it follows that every figure, at its birth, comes forth from a circular figure, and when it ends, it ends in a circular figure.

3. We said that no figure is as full as the circular one. Hence, it follows that the circular figure comes before all other figures.

4. We said that because a pentagonal figure has broader angles than a square one, it is more similar to a circular figure than to a square one. Hence, it follows that in the circle, the pentagonal figure contains more than the square figure, and the hexagonal figure contains more than the pentagonal figure.

5. We said that none of the general figures is as difficult for the imagination to visualise as the circle. Hence, it follows that the imagination can visualize angles more strongly than it can visualize circles.

6. We said that the circular, square and triangular figures are the only general figures. Hence, it follows that the succession of generation and corruption is from the circular figure to quadrangulation and from the square to circulation, given that, the circle is the first figure, the square is the second and the triangle is the third.

7. We said that no figure could exist without the circle. Hence, it follows that beyond the supreme circle of heaven, there is, by nature, no figure.

8. We said that nothing represents the forms and differences of substances as well as a figure. Hence, it follows that the visual power is closer to the power of imagination through figure, than any other sensitive power.

9. We said that figure is a likeness of form just as surface is a likeness of matter. Hence, it follows that science arises in the power of sight earlier than in any other sensitive power, given that form and surface appear more to the visual power than to any other power.

10. We said that without proportion among its parts, no figure could be beautiful. Hence, it follows that the imagination imagines corporeal beauty better through the visual power than through the other sensitive powers.

Quantity

Principles of Quantity

1. Quantity is what makes finite things finite.

2. If there were no quantity, all finite things would be infinite.

3. All corporeal quantities belong to one general quantity.

4. General quantity is both outside and within every corporeal substance.

5. The quantity of surface is a general likeness and figure.

6. Due to simple quantity, substance can exist in its simple and real parts.

7. The quantity of number is a likeness and figure of real quantity.

8. The quantity of figure is in surface and color, and the quantity of substance is within substance.

9. With difference, quantity is the instrument of division.

10. Because a line is made of points, it is quantitatively divisible.

Corollaries to the Principles of Quantity

1. We said that quantity is what makes finite things finite. Hence, we conclude that substance signifies infinite being better through major quantity than through minor quantity.

2. We said that if there were no quantity, all finite things would be infinite. Hence, it follows that there can be no quantity in infinite being.

3. We said that all corporeal quantities belong to one general quantity. This leads us to conclude the existence of one general body to which all bodies belong.

4. We said that general quantity is both outside and within every corporeal substance. This leads to the conclusion that the world is entirely full.

5. We said that the quantity of surface is a general likeness and figure. This leads to the conclusion that every corporeal quantity of corporeal substance has a neighboring quantity.

6. We said that through simple and compound quantity, substance could exist in its simple and real parts. This leads to the conclusion that all compounds are made of simple parts.

7. We said that the quantity of number is a likeness and figure of real quantity. This leads to the conclusion that the imagination can do more with geometry than with arithmetic.

8. We said that the quantity of figure is in surface and color, and the quantity of substance is within substance. This leads to the conclusion that there is an inner quantity as well as an outer quantity.

9. We said that with difference, quantity is the instrument of division. This leads us to conclude that division first arises in the separation of one point from another.

The Center

Principles of the Center

1. The center is the point in which lines converge the most closely.

2. The center is the place where lines reach their greatest perfection.

3. Nowhere is the point as perfect as in the center.

4. A figure cannot be without a center.

5. Not all centers are located here below.

6. Gravity seeks one center and levity seeks another.

7. There is no locus as naturally desirable as the center.

8. Every center is common.

9. One who destroys the center also destroys motion.

10. All the secondary acts of powers are likenesses of the center.

Corollaries to the Principles of the Center

1. We said that the center is the point in which lines converge the most closely. Hence, it follows that the center is round.

2. We said that the center is the place where lines reach their greatest perfection. Hence, it follows that the diametrical line is closer to perfection than any other line inside the circle.

3. We said that nowhere is the point as perfect as in the center. Hence, it follows that a round point is always closer to perfection than a long one.

4. We said that a figure could not be without a center. Hence, it follows that every figure has an appetite for the center, and cannot exist without this appetite.

5. We said that not all centers are located here below. Hence, it follows that the sun, as well as the other planets, are the centers of their respective spheres.

6. We said that there is no locus as naturally desirable as the center. Hence, it follows that all points of angles have an appetite to move.

7. We said that the center is the place where the points of angles repose. Hence, it follows that all points of angles have an appetite to move.

8. We said that every center is common. Hence, it follows that some point is a genus that contains species.

9. We said that one who destroys the center also destroys motion. Hence, it follows that some center is immovable.

10. We said that all secondary acts are likenesses of the center. Hence, it follows that every secondary center has moved.

Capacity

Principles of Capacity

1. Capacity is what enables nature to receive and contain the things that can be received and contained.

2. Space and concavity are the principles of capacity.

3. No point that has the capacity to contain an angle is without quantity.

4. The point's capacity is entirely due to the circle or the angle.

5. All right angles have capacity for containing width, and all acute angles have capacity for containing length.

6. No line can have the capacity to contain an angle without dividing itself into parts.

7. No "now" is capable of division.

8. Because the circle has more concavity than the square or triangle, the circle has greater containing capacity than the square or the triangle.

9. In the generation of things, in the fusion of metals, in the mixture of water with wine, and of flour with water, parts have the capacity to be within one another.

10. If parts had no capacity to be within each other, there could be no true mixture or succession.

Corollaries to the Principles of Capacity

1. We said that capacity is what enables nature to receive and contain the things that can be received and contained. Hence, we can conclude that in nature there is space for circles and angles.

2. We said that space and concavity are the principles of capacity. Hence, we can conclude that surface and capacity have the same end.

3. We said that no point that has the capacity to contain an angle is without quantity. Hence, we can conclude that some point has quantity.

4. We said that the point's capacity is entirely due to the circle or the angle. Hence, we can conclude that in some point there is concavity.

5. We said that all right angles have capacity for containing width, and all acute angles have capacity for containing length. Hence, we can conclude that continuation is due to width more than to length.

6. We said that no line could have the capacity to contain an angle without dividing into parts. Hence, we can conclude that there is no line without points.

7. We said that no "now" is capable of division. Hence, we can conclude that time is not made of parts.

8. We said that because the circle has more concavity than the square or the triangle, the circle has greater containing capacity than the square or the triangle. Hence, it follows that the circle contains more with a shorter line than do the square or the triangle.

9. We said that in the generation of things, in the fusion of metals, in the mixture of water with wine, and of flour with water, parts have the capacity to be within one another. Hence, we conclude that corporeal essences have reciprocal containing capacity.

10. We said that without the capacity of parts to be within each other, there could be no true mixture or succession. Hence, we can conclude that one compound capacity can be made of several simple capacities.

Length

Principles of Length

1. Length is a figure made of narrow points.
2. Length is always more similar to form than to matter.
3. Division always begins with a long line before a wide line.
4. Curved length is always made of curved points.
5. Vertical length is always more natural than transversal length.
6. Vertical length is always thinner at the top than at the bottom.
7. Fullness consists of length, width and depth.
8. There can be no length without width and depth.
9. A line is always more similar to length than to width or depth.
10. There is no length without termination.

Corollaries to the Principles of Length

1. We said that length is a figure made of narrow points. Hence, we can conclude that width is a figure made of wide points and roundness is a figure made of round points.

2. We said that length is always more similar to form than to matter. Hence, we can conclude that matter naturally requires width.

3. We said that division always begins with a long line before a wide line. Hence, we can conclude that in division, form has action and matter has passion.

4. We said that curved length is always made of curved points. Hence, we can conclude that circles and angles are made of long and curved points.

5. We said that vertical length is always more natural than transversal length. Hence, we can conclude that the natural

transversal line is made of points tempered by lightness and heaviness.

6. We said that vertical length is always thinner at the top than at the bottom. Hence, we can conclude that trees are thick at the bottom due to heavy points, and slender at the top due to light points.

7. We said that fullness consists of length, width and depth. Hence, we can conclude that length, width and depth are parts of fullness, and that every part is in every other part, and consequently, every point is in every other point, since points are parts of length, width and depth.

8. We said that there could be no length without width and depth. Hence, we can conclude that in a full body, points are within one another.

9. We said that a line is always more similar to length than to width or depth. Hence, we can conclude that in generation, lengthwise growth begins before growth in width and depth.

10. We said that there is no length without termination. Hence, we can conclude that the utmost terminus of length is made of a long point.

Width

Principles of Width

1. Width is an extension made of wide points.

2. Width proceeds from the temperament of light and heavy points.

3. Wider points make wider leaves.

4. If there were no width, only the circle would be a figure.

5. There can be no width without angles.

6. The width and weight of clouds produce wind here below.

7. Color and surface develop through width.

8. Width always stands naturally between length and depth.

9. All temperament of air consists more of width than of length or depth.

10. Breadth makes the difference between high and low.

Corollaries to the Principles of Width

1. We said that width is an extension made of wide points. Hence, we can conclude that the more a point extends in width, the less length and depth it has.

2. We said that width proceeds from the temperament of light and heavy points. Hence, we can conclude that clouds are made of wide points.

3. We said that wider points make wider leaves. Hence, it follows that air, which is more extensible than the other elements, has points that are wider than those of the other elements are.

4. We said that if there were no width, only the circle would be a figure. Hence, it follows that the square and the triangle are figures that proceed from angles.

5. We said that there could be no width without angles. Hence, it follows that all angles are extremities of figures.

6. We said that the width and weight of clouds produce wind here below. Hence, it follows that the matter constituting the wind's width is not as heavy as the matter of clouds.

7. We said that color and surface develop through width. Hence, it follows that color and surface are made of wide and extended points.

8. We said that width always stands naturally between length and depth. Hence, it follows that as cold contracts air in width, air's matter grows longer and deeper.

9. We said that the temperament of air always consists more of width than of length or depth. Hence, it follows that the points of air are wider in the spring than at any other time of year.

10. We said that width makes the difference between high and low. Hence, we can conclude that high divides from low by a wide point.

Depth

Principles of Depth

1. Depth is in the region of the center.

2. The acute angle is the principle of depth.

3. In every point in which there is an angle, there is depth.

4. Without depth, no part can be in another part.

5. Length and width are not as well disposed as depth to mixture and composition.

6. Depth disposes parts to enter into each other, just as width disposes them to expand, and length disposes them in sequence.

7. In the center of a square, where four diametrical lines meet, there must be 4 deep angles, and at the center of a pentagon where 5 diametrical lines meet, there must be 5 deep angles.

8. Melancholy and depth are concordant.

9. No element has an angle as deep as earth's angle.

10. All the sensitive powers have more virtue for sensing depth than for sensing length or width.

Corollaries to the Principles of Depth

1. We said that depth is in the region of the center. Hence, it follows that appetite reposes in the center more in depth than in length or width.

2. We said that the acute angle is the principle of depth. Hence, it follows that depth is more consistent with the triangle than with the square.

3. We said that in every point in which there is an angle, there is depth. Hence, it follows that every point that has depth has discrete quantity.

4. We said that without depth, no part could be in another part. Hence, it follows that depth is the disposition, or gateway allowing parts to be within each other.

5. We said that length and width are not as well disposed to mixture and composition as depth is. Hence, we conclude that depth is the primordial gate of generation.

6. We said that depth disposes parts to enter into each other, just as width disposes them to expand, and length disposes them in sequence. Hence, it follows that depth is an accident and not a substance.

7. We said that in the center of a square, where four diametrical lines meet, there must be four deep angles, and at the center of a pentagon where five diametrical lines meet, there must be five deep angles. Hence, it follows that every center has in itself one indivisible point that is the subject of many angles.

8. We said that melancholy and depth are concordant. Hence, we conclude that memory is the depth of science.

9. We said that no element has an angle as deep as earth's. Hence, it follows that in the earth's center, lines are closer to each other than in any other center.

10. The sensitive powers all have more virtue for sensing depth than length or width. Hence, we conclude that the deepest sense impressions are the greatest.

We have dealt with the principles of Geometry and their corollaries, now we intend to deal with questions that you can make and solve with these principles and their corollaries. Because these are natural principles, we apply the questions to natural subjects.

Book 2, Part 3 - questions, & resolution of some doubtful issues in Geometry

In this part, we intend to make questions, and before answering each question, we will raise an objection. We propose to make 100 questions and in solving them, we will refer to the principle in question as well as to the corollary of the principle, and then we will respond to the form of the objection. Here we intend to follow the same process throughout the entirety of this part.

Points

Questions about Points

Question 1

Is the point, or the line the first most basic ingredient of composition?

Objection

If the point, and not the line, were the most basic ingredient of composition, it would follow that a line is composed of discrete points and cannot be continuous or full.

Solution

Go to the first principle of points with its corollary, which indicates the answer by saying that a point is part of a line mediating between the point and the subject composed of lines. Hence, the point must be the first simple ingredient of composition, or else, composition would not be composed of prime simple ingredients, which is impossible. Moreover, the objection that if the point were the first simple ingredient of composition, then the line would be made of discrete points and could not be full is not true. Now in a line, every point is in every other point just as each simple element is in the others, and composition results from their mixture. Fire enters into air and gives it its heat, but fire's heat does not abandon its own subject. Rather, it enters with it, and likewise with the other elements, so that in mixture, the line is continuous throughout the length, width and depth of the body.

Question 2

In nature, is there a common point?

Objection

In nature, there cannot be any common point, because if there were such a point, it would be made of parts, and mixture would be both common and not common, given that all common things are common due to their parts.

Solution

Go to the second principle of points and its corollary that indicates the answer to the question. Now your objection would be true if all common points were common due to the compound parts of a common point. Nevertheless, a simple and common point can be common although it has no parts or quantitatively distinct parts in lines, loci and figures. Such is the simple form of fire inasmuch as it is one indivisible point common to all compound and secondary forms of fire, and to their matter.

Question 3

Are surfaces made of points, or lines?

Objection

No surface can be made of points, given that a point has no length, width or depth. Because there cannot be any surface without length, width and depth, it follows that surfaces are made of lines.

Solution

Go to the third principle of points, and when you say that surfaces cannot consist of points, you are merely referring to simple points, given that simple points have no length, width and depth. However, this is not true of compound points, given that all compound points have length, width and depth.

Question 4

Does corporeal nature contain indivisible points?

Objection

In corporeal nature, which is continuous throughout all of its subjects, there is no vacuum. However, there would be a vacuum if corporeal nature contained some indivisible point of its own essence, because such a point cannot divide into a quantity of discrete parts.

Solution

Refer to the fourth principle, where the solution is indicated; and you would be right if the point you mentioned had length, width and depth; but division cannot move this kind of point to generation and corruption in elemental nature.

Question 5

Can you divide a point that is the subject of an angle?

Objection

You cannot divide a point that is the subject of an angle, because if you could, it would follow that the angle would have no subject and no matter in which to exist.

Solution

Look up the corollary to the fifth principle. You would be right if the point of the angle were not a body and a compound. but it is a body, and as such, it is the compound subject of an angle, so you can divide it into its components.

Question 6

Could some sensible point be indivisible?

Objection

A point made with a needle, and a very tiny colored point can be sensed, but remain indivisible because they have no long, wide and deep lines.

Solution

Go to the corollary of the sixth principle; and because any point made with a needle is full of air, which is compound and divisible, and because a colored point is a compound body with a compound surface, your statement is not true.

Question 7

Is there a general point in nature?

Objection

If there were a general point in nature, it would have to have specific points, and the specific points would have to have individual points, which would mean that the general point and its species would have to be real, but since genus and species are not real but only intentional things, there can be no general point in nature.

Solution

Go to the seventh principle, and you are wrong in saying that no general point can really exist in nature, because prime matter, which is a point general to all specific points, is a real thing, as are natural goodness and other natural principles like one color, or whiteness. So likewise, the point at the center is general to the points in the angles, a point in a right angle is more general than a point in an acute angle, and every one of these generalities is a real thing.

Question 8

In nature, is there any point so simple that it has nothing else in itself but its own pure essence?

Objection

There can be no simple point in nature, because in any point there must be several things, such as locus, quantity, fullness etc.

Solution

Go to the eighth principle. You are right in saying that there must be several essences in any point, given that many essences go into composing a point. However, if you consider a point as a simple essence inasmuch as it is what it is, and not some other essence, there can be single, simple points in nature, like one color, one corporeity etc.

Question 9

In the supreme surface of heaven, is there some point that is not common?

Objection

Any point at the surface of a line must be common, because if it were not common, it could not terminate a line; and because the surface of heaven is made of points, and as it is the terminus of the supreme line, this surface cannot have any point that is not common.

Solution

Go to the ninth principle. In saying that no point can terminate a line unless it is common, you are right with regard to the nature of the points under the supreme surface of heaven. Nevertheless, heaven must terminate, because a body can only be finite. Therefore, the points making up the supreme surface of heaven must be of a nature not shared by the points inside. Otherwise, it would be possible for heaven, or some other body to be infinite inasmuch as there would be an infinite process where each point participates with another one above it in an endless succession of points, which is impossible. Therefore, the topmost points must be of a nature not shared by the points below. Due to this nature, their commonness must finish at the top, so that heaven can have a final termination.

Question 10

Can a point as such move from place to place?

Objection

No point as such can move from place to place, given that a point cannot move without a line.

Solution

Go to the tenth principle, and you are right inasmuch as a point cannot move without a line just as form cannot move without matter; but you are wrong inasmuch as a line can move a point from place to place just as the motion of a boat moves a sailor from place to place.

Lines

Questions about Lines

Question 1

Since lines are essential to figures, can lines terminate figures?

Objection

A line cannot be terminate and enclose a figure, for if it could, then an essence could terminate itself, which is impossible.

Solution

Go to the first principle. You are right in saying that an essence does not terminate itself. Nevertheless, a line, although it is essential to a figure, terminates the figure just as a point terminates the line of which it is a part. Therefore, a line terminates in itself at some point, just as the whole terminates in its parts and the parts terminate in their whole.

Question 2

Does a line have matter?

Objection

A line has no matter, for if it had any matter, it would be a body, and one body would be within another, because a line exists in a figure that has a body.

Solution

Go to the second principle, and in saying that a line has no matter, because if it had any matter it would be a body, you would be right if it had its own matter essentially distinct from the body's matter. However, a human shape, for instance, is comprised of lines and human essence, and the man's matter is something common to his shape and its lines.

Question 3

Does a surface have matter?

Objection

A surface has no matter, because if it had any, it would be a body, not an accident.

Solution

Go to the third principle of lines. You are mistaken in saying that surface has no matter because it would be a body if it did. Now the contiguousness of lines is an accident that converts with the surface of the body that makes up the substance of the surface consisting of lines made of points which have a corporeal essence and which constitute and compose the body with lines.

Question 4

Is a line full of anything?

Objection

A line is not full of anything, as it is a first basic ingredient that fills a figure.

Solution

Go to the fourth principle of lines. You are right in saying that the line is a first basic ingredient that fills a figure, but it could not fill it unless it was itself full of something.

Question 5

Given that a line is long by nature, what gives it its natural width?

Objection

A line is naturally wide when it is full of wide points.

Solution

Go to the fifth principle. You are right in saying that a wide line is naturally made of wide points, inasmuch as a surface is, by nature, made of wide points.

Question 6

Which line is the fullest?

Objection

No line is as full as a straight line, given that straightness is more consistent with a line than obliquity is.

Solution

Go to the sixth principle of lines that indicates that the more continuous a line is, the fuller it is. Now as there is no break or division in a circular line, it is fuller than a straight line. Moreover, when you say that straightness suits a line better than obliquity does, you are right with regard to lines that do not belong to spherical or round bodies, which are more mobile than bodies that are not round.

Question 7

Can there be an infinite line?

Objection

God has infinite power, a line made of points can extend point by point to infinity, and thus God can make, or create one point after another, so that an infinite line can exist by God's power and will.

Solution

Go to the seventh principle of lines. You are right in saying that God has infinite power. However, because no compound thing made of finite parts can have an infinite nature, even though God with his supernatural power could create an infinite line, nevertheless, a line can only be finite by nature. God cannot cause infinite growth from all eternity; he cannot do this because the subject cannot receive it.

Question 8

With which line does nature most express its appetite?

Objection

There is no line as continuous and full as a circular line, and therefore a circular line must express the greatest natural appetite.

Solution

Go to the eighth principle. You would be right if power and object could meet as directly through a circular line as through a diametrical line.

Question 9

Does a line occupy space?

Objection

A line occupies no space, given that a line is not a body, and without a body, it cannot occupy space.

Solution

Go to the eighth principle. You are right in saying that a line is not a body. Nevertheless, inasmuch as a line is made of points, it is a part of a body, and inasmuch as it is a part of a body, one line occupies another line's place, as a black line placed over a white line occupies the place of the other line inasmuch as it does not participate with the white surface.

Question 10

Is the influence shed by heaven on things below disposed in a circular line, a square line, or a triangular one?

Objection

Because stars have spherical and round bodies, they influence things below with an influence that is round like a circle.

Solution

Go to the tenth principle of lines. Now what you say is right. But because the square line stands naturally between the circular line and the triangular line, the influence first comes in a circular figure, then in a square one, and because the influences incline towards loci proper to their specific natures, they change into a triangular habit, as indicated earlier in the figure of the planets.

Angles
Questions about Angles

Question 1

Are angles of the essence of squares and triangles?

Objection

If angles were of the essence of squares and triangles, they would both terminate and not terminate in squares and triangles, so that squares and triangles would be figures without termination, which is impossible.

Solution

Go to the first principle of angles. You would be right if angles did not have common points whereby common angles can be of the essence of squares and triangles inasmuch as they partake of points, like the points in the circumference and the center to whose essence the angles belong

Question 2

Why does a square contain as much with short lines, as does a triangle with long lines?

Objection

The reason why a square contains as much with short lines as a triangle does with long ones, is that a square, by nature, has four angles whereas a triangle has only three.

Solution

Go to the second principle of angles, where this question is solved. You would be right in a secondary sense, inasmuch as a square with its right angles can contain more in itself than a triangle with its acute angles. In addition, because a square has right angles, it naturally has four angles, and therefore, with regard to containing capacity, it follows that a square contains more within itself with short lines than does a triangle because it has four angles, whereas a triangle has only three.

Question 3

Is an obtuse angle more similar to a triangle than to a circle or a square?

Objection

An obtuse angle partakes of the circle, the square and the triangle, and because the circle is the first figure, and a nobler figure than the square or the triangle, an obtuse angle is closer and more akin to the circle than to the triangle or the square.

Solution

Go to the third principle. You are right in saying that the circle is the first figure, and nobler than the square or the triangle. However, it does not mean that an obtuse angle is closer to the circle than to the square or the triangle, given that the circle has no angles, whereas the square and the triangle have angles. Because the square stands naturally in the middle between the circle and the triangle, as we proved, an obtuse angle is more similar to the square than to the circle or the triangle.

Question 4

When an angle of a body is corrupted, does the matter of this angle go over to another square, or does it recede into a round body?

Objection

Whereas an angle in actual existence is more like an angle in potentiality than like a circle that has no angles, a corrupted angle must naturally recede to an angle and not to a circle.

Solution

Go to the fourth principle of angles. You would be right if natural generation began bringing back individuals of species through the square, and not the circle. This cannot happen because in generation, there must first be confusion and circulation of the elements within each other, and from one quantity into another, producing four special dispositions in angles and straight lines. The eye can see this in a plant, for when its seed decays in the earth and generates another plant, it makes a round figure before making a long one, and afterward, the round figure makes a long figure through the roots and trunk of the plant as it grows in length. This is visible when trees make circular figures with their buds before making long ones.

Question 5

Does the surface of a spherical body consist of angles?

Objection

In a spherical body, there are no angles, given that its figure is circular, and it could not be circular if its surface were composed of angles.

Solution

Go to the fifth principle, and you are right with regard to concave circular figures, like the figure of the white circle. Nevertheless, inasmuch as a spherical body is elemented and full, you are wrong, given that a surface is made of lines and lines of points and in every surface composed of points and lines there must be breadth, or width that makes long and transversal lines that are long and transversal across the width of the surface.

Question 6

Is the center of an angle a simple point, or a compound one?

Objection

The center of an angle cannot be a compound point, and it must be simple, because if it were a compound and not a simple point, it could not be the subject of the angle's figure.

Solution

Go to the sixth principle, and you are right in saying that the center of an angle could not be the subject of the angle's figure if it were not a simple point. Nevertheless, this does not mean that it is not a compound point, given that it is simple inasmuch as it is made of simple principles, and it is a compound point inasmuch as the principles enter into composition.

Question 7

Is an angle made of things that are convertible, or non-convertible?

Objection

Convertible things are those that convert through proper and appropriated qualities, like fire and air, which convert into heat. Moreover, non-convertible things cannot convert because of their discordant proper qualities, like fire and water, which cannot naturally convert through heat and cold. Therefore, angles must be made of non-convertible parts.

Solution

Go to the seventh principle. You are right in saying that angles are made of non-convertible points, which cannot properly convert through the motion of simplicity. For instance, water heated by fire has the figure of heat due to the conversion of cold and moisture into the figure of heat. Similarily, an angle is made of convertible points inasmuch as many simple points go into making up one compound point, which is the center of the angle.

Question 8

Is there an acute point in a right angle?

Objection

There can be no acute point in a right angle, because if there were one, the angle would be of a triangular, not quadrangular nature, which is impossible.

Solution

Go to the eighth principle, and you are wrong in saying that there is no acute point in a right angle, given that there is a potential acute point in a right angle, through which point the acute angle of the triangle potentially exists in the right angle. However, you are right with regard to the actual acute angle. Here, we realize that there is more truth in art than in the senses. Now an acute angle appears to be actually in a right angle, in which it does not exist actually but only figuratively. If it actually existed in the right angle, it would follow that right and acute angles are naturally convertible, which is a contradiction.

Question 9

Does the acute angle belong to the essence of the right angle?

Objection

The acute angle cannot be of the essence of the right angle, because if it were, it would always remain in the species of the right angle.

Solution

Go to the ninth principle. You are wrong in saying that the acute angle would remain in the species of the right angle if it were of its essence. Now a line dividing a right angle by entering into the corner of a square produces an acute angle from the essence of the right angle. Similarily, oats are of the essence of barley turned into another species, which is oats and not barley.

Question 10

When two lines cross the center of a circle, is the center actually and entirely round?

Objection

The center of a circle crossed by two diametric lines cannot be round because of the angles made by the lines.

Solution

Go to the tenth principle. You are right inasmuch as two or more diametric lines touch and cross each other in the center, but you are wrong inasmuch as the center is actually round even before the lines meet in it.

Figures

Questions about Figures

Question 1

What are the prime principles of figures?

Objection

The prime principles of figures are quantity and color.

Solution

Go to the first principle of figures, which clarifies what the prime principles of figures are, and both quantity and color are material and secondary principles of figures, given that figures consist of the circle, triangle and square.

Question 2

When a figure decays and is gone, in what habitus does it end up? If it has a final habitus, is it square, circular or triangular?

Objection

No point is as close to non being as an acute point, and because lines are made of points and figures are made of lines, the final resting habit of a figure must be triangular, which is closer to an acute point than to any other point.

Solution

Go to the second principle. You are right in saying that the triangle is more similar to an acute point than to any other point, but generation and corruption proceeds through relative contrary and concordant natural qualities of the elements. This is why the ultimate habit of a corrupted figure is a round point.

Question 3

Since a circular figure is fuller than a square figure or a triangular figure, of what is the circular figure more full than the square or the triangular figure?

Objection

That which fills a circular figure more than a triangular or square one is the continuation of air, which is more continuous in a circular figure than in a square or triangular one.

Solution

Go to the third principle of figures, which indicates that a circular figure is fuller than any other figure as it has more round points than any other kind of points. Due to this, a circle has no beginning or end. Moreover, you are right with regard to matter, i.e. the matter of what is contained in a circular figure, but you are wrong with regard to form.

Question 4

Why does a pentagonal figure contain more than a square figure?

Objection

The reason why a pentagonal figure contains more than a square figure is that a pentagon has five angles, while a square has only four.

Solution

Go to the fourth principle, which indicates that the reason why a pentagonal figure contains more than a square figure is that it is made of wider angles, and this is the primary and formal cause, whereas the other reason is secondary and material.

Question 5

Why is a circular figure more difficult to visualize than a figure with angles?

Objection

The reason why a circular figure is more difficult to visualize than a figure with angles is that it is a more general figure.

Solution

Go to the fifth principle, and you are right with regard to matter, but not with regard to form. Given that a circular figure has no beginning or end, you canot visualize it as easily as a figure with a beginning, middle and end.

Question 6

Of what does the first sucesion in generation and corruption consist?

Objection

In generation and corruption, the first succession is in the parts removed from the subject that corruption reduces to non-being, and that are given to a subject that comes into being through generation.

Solution

Go to the sixth principle, and you are right with regard to matter, but not with regard to form.

Question 7

Can nature allow another heaven to exist outside the supreme heaven, so that neither heaven is within the other?

Objection

Since the nature of the heaven within which we exist would not be in the other heaven, it could not impede the other heaven from existing.

Solution

Go to the seventh principle where it says that there cannot be any triangle or square outside heaven. Now if there could be two heavens, each existing outside the other, like the Sun and the Moon, so that both are outside each other and neither one is in the other, their circles would make a figure outside themselves, like two eggs touching each other. However, this figure would be void, as it would contain nothing within itself, but such an empty figure is impossible.

Question 8

Which sensitive power is the closest to the imagination?

Objection

None of the senses has as great an act of sensing as the sense of touch, as can be sensed by touching hot iron or boiling water, or in sexual intercourse, or a bodiy wound. And for this reason, the sense of touch, whose act is greater than that of the other senses, must be closer than any other sense to the imaginative power, given that all the things that the imaginative attains through the sensitive really exist.

Solution

Go to the eighth principle, and you are right with regard to the act of sensing by touch, but the imaginative imagines things most strongly trough visualization, given that sight attains color, circular lines and angles.

Question 9

Through which sense does science first arise?

Objection

In no power of sense does science arise as readily as in hearing, which is evident to the senses because teachers show more things to students through hearing than through sight.

Solution

Go to the ninth principle. You are right with regard to what the intellect receives from the imagination, but you are wrong with regard to what the imagination receives from the senses.

Question 10

Through which sense does the imagination naturally first perceive beauty?

Objection

The greatest sense impressions come from taste and touch, and therefore the imagination must perceive things naturally through taste and touch before any of the other senses.

Solution

Go to the tenth principle, and you are right with regard to the purpose for which man wants to touch, or eat. Nevertheless, you are wrong with regard to the first natural act, for instance, a man first imagines the beauty of the beautiful shape he wants to have, and he imagines white bread before wanting to eat some.

Quantity

Questions about Quantity

Question 1

Does a major quantity of virtue than signify God better than a line of major quantity does?

Objection

God is virtue, and not a line, and He is infinite. For this reason, a major quantity of virtue signifies God better than does a line of major quantity.

Solution

Go to the principle of quantity, and you are right with regard to the signification of an intellect stripped of all imaginable species. but you are wrong with regard to imagination and intellect, given that the intellect, when it imagines a major line, considers the infinite extension of divine substance free of any quantity.

Question 2

If there were any quantity in God, would this quantity make Him finite?

Objection

God is an infinite being, and if there were any quantity in God, He would be finite only with regard to this quantity, but infinite inasmuch this quantity does not entirely pervade His essence.

Solution

Go to the second principle, and you are wrong, given that God is a simple, indivisible substance, which He would not be if He were finite due to some natural quantity, while remaining infinite due to another essence free of quantity.

Question 3

Is there one general quantity that gives rise to all other quantities?

Objection

It is obvious that between the quintessence and the four substances of the world, there is a natural division in space, whence it follows that not all quantities spring from one quantity.

Solution

Go to the third principle, and you are right in saying that there is a division between the quintessence and the four substances of the world. However, you are not right with regard to the continuation and succession of the undivided line, or with regard to the prime essence of the world. Although it is one because of the unity of general essence, it nevertheless manifests through many species of essences. For instance, wine and oil divide into discrete quantities, but remain under one quantity that is common to both, and continuous through the quantity of prime matter.

Question 4

Is the whole world entirely full?

Objection

The whole world cannot be full, because if it could be, bodies would be within each other, which is impossible.

Solution

Go to the fourth principle, and you are right in saying that bodies cannot exist within each other, as one body occupies one locus and another body occupies another locus. For instance, a golden cup cannot be in the same locus as a silver cup, while each cup retains its own configuration and shape. However, by melting both cups down to make one vase with one shape, the body of the golden cup can be within the body of the silver cup and vice-versa, like one part within another, where both parts compose one quantity.

Question 5

Can the imagination imagine general quantity?

Objection

If the imagination could not imagine general quantity, it could not imagine specific quantity.

Solution

Go to the fifth principle, and you are right with regard to subalternate general quantity, but wrong with regard to entirely general quantity, like the quantity of heaven and of the continuous lines of its center and diameter, which the senses cannot perceive.

Question 6

Are all compounds made of simple parts?

Objection

In elemented things, no compound is made of simple parts, for if it were made of simple parts, each part would remain in its own specific simple quantity, and the compound would have no continuous quantity, so that it would be both a compound and not a compound, which is impossible.

Solution

Go to the sixth principle. Now you would be right if one simple part could not be within another simple part. For instance, one simple element could not be in another simple element, where it can be due to the motion of general mixture, in the generation of elemented things, and in gold and silver melded into a coin, as we said earlier.

Question 7

Is arithmetic a subject as proper to the imagination as geometry?

Objection

Because an arithmetician attains numerical quantity more easily than a geometer attains corporeal quantity, the imagination has more virtue in things it attains easily than in those it attains with difficulty.

Solution

Go to the seventh principle, and you are right with regard to the ways in which we see and hear things, but not with regard to the way in which we imagine them, given that arithmetic does not have circles and angles as a subject.

Question 8

Does the selfsame same quantity exist within and outside substance?

Objection

Quantity within substance cannot exist outside substance, nor can external quantity be inside, for if this were possible, it would follow that what is inside is identical to what is outside, which is impossible.

Solution

Go to the eighth principle. You are right inasmuch as quantity, color and shape are quantities distinct from the inner quantity of substance. However, you are wrong inasmuch as these distinct quantities make up one continuous compound quantity which compre-hensively measures all extremes of substance.

Question 9

What is the prime principle of the division of substance?

Objection

The prime principle of the division of substance is corruption

Solution

Go to the ninth principle, and you are right with regard to material division, but wrong with regard to formal division, which is made first by dividing form.

Question 10

Is substance divisible before quantity?

Objection

Obviously, substance is divisible by the division of quantity. Therefore, quantity divides before substance does.

Solution

Go to the tenth principle, and although the division of quantity divides substance, it does not mean that division is in quantity before substance, given that quantity cannot be without a subject. Moreover, the division of its parts from one another is what divides substance, whereas quantity divides by accident, and is consequent to the division of substance.

The Center

Questions about the Center

Question 1

Is the center of a circle naturally round?

Objection

It is obvious that nature does nothing against itself, for if it did, it would not be what it is, but rather something above the course of nature, which is impossible. Moreover, because in the center of a pentagon inscribed in a circle there are potential acute angles through continuous lines from the center to the angles of the pentagon, it is obvious that the center of a circle is not naturally round, because if it were, it could not naturally receive the acute angles of the pentagon.

Solution

Go to the first principle. If what you say were simply right, there could be no center in the middle of a circle, which would not be round.

Question 2

Why is the diametric line closer to perfection than any other line?

Objection

The reason why the diametric line is closer to perfection than any other line is that it divides a circle into equal parts.

Solution

Go to the second principle, which indicates that because a diametric line is the one that can best participate in its own line, it is closer to perfection in the center and due to the center than because of anything else. Although it is true that a diametric line is closer to perfection because it divides a circle into equal parts, it owes its perfection even more to its participation with many lines at the center.

Question 3

Why is the center, by nature, more desirable than any other locus?

Objection

The reason why the center is more desirable than any other locus is that it stands in the middle of a circle.

Solution

Go to the third principle, and you are right in saying that a point is closer to perfection in the center than in any other place. Now due to this perfection, the center is more desirable than any other locus. Moreover, there is more repose in the center than there is anywhere else.

Question 4

Why can a figure not be without a center?

Objection

The reason why a figure cannot be without a center is that in every figure there is a locus in the middle that is its center.

Solution

Go to the fourth principle, and you are right in saying that in every figure there is a locus in the middle that is its center. However, every figure has an appetite that moves it to seek the center, which appetite it would not have without a center. Moreover, it could not exist without this appetite. This is why a figure cannot be without a center.

Question 5

Where is the center of the Sun?

Objection

Since every center is located in the middle, the center of the Sun's sphere must be located in its middle.

Solution

Go to the fifth principle, and you are right with regard to the center of a spherical body, but wrong with regard to a concave body, like the solar sphere as well as the other planetary spheres, which are concave so that the spheres can contain one another. Hence, the sphere of Saturn contains the sphere of Jupiter; the sphere of Jupiter contains the sphere of Mars; the sphere of Mars contains the sphere of the Sun; the sphere of the Sun contains the sphere of Venus; the sphere of Venus contains the sphere of Mercury, and the sphere of Mercury contains the sphere of the Moon. This is the way it must be, so that they do not lose their shape and leave a void. They would leave a void if they were spherical bodies, as mentioned earlier, like two or more eggs touching each other and making a non-circular figure.

Question 6

Does the center have any appetite for motion?

Objection

In a very heavy stone, there is a center, and this center wants to move toward the nether center, due to the magnitude of its weight.

Solution

Go to the sixth principle, and you are right in saying that a falling stone has an appetite for moving downward, and this is due to the general center of water and earth, which is the lowest, and attracts all the other centers belonging to it.

Question 7

Do the centers of angles have any appetite to move?

Objection

Since the square and the triangular figure have perfect being due to their angles, the center points of their angles have no desire to move.

Solution

Go to the seventh principle. You are wrong in saying that the centers of the angles in squares and triangles have no appetite to move, because all triangular and square figures are corruptible.

Question 8

What are the species of the common center?

Objection

The common center is a point in a body that is in the middle of a circle, and its species are the angles it potentially holds, inasmuch as it can be the terminus of many lines.

Solution

Go to the eighth principle, which indicates that the centers of the angles in squares and triangles are species of the center of the circle, because all other figures arise from the circle; and you are wrong, because potential angles are not actual things.

Question 9

Which center is the one that does not move?

Objection

There cannot be any unmovable center. Every center is a creature and as such, every center moves.

Solution

Go to the ninth principle, which indicates that the center that does not move is the center of the earth. In addition, it would be even better if you said that God is the unmovable center that we desire. Now the center of fire is unmovable inasmuch as it is a general appetite of heat, as are other centers. However, you are right about creation.

Question 10

Heating, cooling, growing, seeing and other such secondary attributes are secondary centers. Hence, we ask whether all these modes of action are in the middle between power and object.

Objection

If heating, cooling, growing, seeing and other such secondary attributes were not in the middle between power and object, they could not be centers, given that a center, by nature, must stand in the middle.

Solution

Go to the tenth principle, and you are wrong inasmuch as all created secondary acts depend, due to the motion of the major end, more on form than on matter, and by reason of this major end, all secondary created acts move from one end to another.

Capacity

Questions about Capacity

Question 1

Of what does capacity consist?

Objection

Capacity is due to the disposition of matter in a body that is disposed to contain something.

Solution

Go to the first principle of capacity with its corollary, and you are right with regard to the substantial act. However, the principle of capacity represents natural spaces that formally result from it. As squares and circles exist in nature, substances or figures have capacity to contain angles and circles. Hence, one body can be contained in another and one part in another, to make up one composition, and linear continuity in substance.

Question 2

Do surface and concavity equally constitute capacity?

Objection

If surface and concavity had one common purpose, it would follow that the supreme surface of heaven would have the capacity to contain some kind of body within its concave space, and so on ad infinitum, which is impossible.

Solution

Go to the second principle, and you are right with regard to the supreme surface, but this question is about surfaces that cover the concavity of bodies, like the sphere of fire whose concavity serves to contain the sphere of air.

Question 3

Does a very tiny point have the capacity to contain another point?

Objection

A point so small that it cannot be any smaller has no capacity to contain another point that is not as small.

Solution

Go to the third principle. Now you would be right, if every point existed actually and fully as something on its own. Nevertheless, you are wrong, inasmuch as points can contain one another due to natural concavity, generation and fusion.

Question 4

In things below heaven, is there any point that is not concave?

Objection

By experience, we see that a grain of wheat or a grain of sand is full, and not concave.

Solution

Go to the fourth principle, and you are right with regard to the fullness of dense bodies, but you are wrong with regard to the natural capacity and concavity that exists in a grain of wheat or sand, as each contains many parts, some of which contain other parts that go into the composition of a grain of wheat.

Question 5

Does capacity owe its existence to long points much as it owes it to wide or round points?

Objection

Capacity and concavity are more suited to long points than to other points, since acute angles are made of long points and have greater concavity with their longer lines.

Solution

Go to the fifth principle, which indicates that capacity is due more to width than to length and to the circle more than to the angle. However, you are wrong, even though long lines are more continuous due to acute angles than due to right angles, given that a right angle contains more in its width than does an acute angle in its narrowness.

Question 6

Does any line have the capacity to divide into points equal to it?

Objection

A line consists of points, but contains no actual points, or else it could be neither continuous nor full, because the configuration and situation of the points would impede its perfect continuity. Therefore, a line cannot divide into points that are actually in it, but into points that are in it potentially.

Solution

Go to the sixth principle. Now you would be right if in the line some points were not within other points. Moreover, they actually exist within each other just as gold and silver melded in a coin. Nevertheless, you are wrong in saying that the constituting points of a line are in potentiality in the line, for if they were, the line would be made of non-actual points, so that it would have nothing for constituting its act, but it would rather have to remain as something potential.

Question 7

Is time made of parts?

Objection

Time must consist of parts, because if it did not consist of parts, nothing could move in time, nor would there be any past or future time.

Solution

Go to the seventh principle. You are wrong in saying that time must be made of parts so that things can move in time from the present to the past and the future. Things move in time as they exist in time like a sailor lying or sitting in a boat, and thus, time moves through the parts of days and hours from one part to another, and due to this motion, past and future time arises.

Question 8

Why does the circle have greater containing capacity than the square and the triangle?

Objection

The circle has greater containing capacity than the square and triangle because it has more concave points than the square or the triangle.

Solution

Go to the eighth principle, and you are right in saying that the circle has more concave points than do the square or triangle if it contains this square or triangle, but if the figures are all mutually equivalent in containing capacity as shown in the plenary figure, each figure has as many concave points as the others.

Question 9

Since wine is a full body, and water is a full body, how can both have the capacity to contain the other in a cup in which they are mixed?

Objection

Wine and water are elemented substances composed of points and lines, and in the cup, one body is close to the other so that the points and lines of both bodies are also close.

Solution

Go to the ninth principle, and you are wrong in speaking of the mixture of wine and water in terms of closeness, because if this closeness did not bring them within each other, neither would the body of the wine and water be full, nor would it have linear continuity.

Question 10

Can one quantity be composed of many simple quantities?

Objection

Quantity is an accident, and an accident cannot be composed of accidents, since in every composition, there must be matter that cannot have any accidents.

Solution

Go to the tenth principle. You are right in saying that composition does not simply arise from accidents. However, as the substances of the accidents enter into composition, their quantities, qualities and other accidents make up common quantities, qualities and so forth, as for instance in the taste of a rose and the taste of sugar that result in one taste, and likewise with other such things.

Length

Questions about Length

Question 1

Is a circle made of round, or long points?

Objection

A circle is a long line that contains its extremities in itself, and so it must be made of long points.

Solution

Go to the first principle, and you are right in saying that a circle is a long line, but inasmuch as it makes a round figure, it must be made of round points so that its outer surface can be round, which it could not be with flat, long points.

Question 2

Why does a long point signify form more than matter?

Objection

The reason why a long point signifies form more than matter is that form exists in the extremities of the matter which fills with round points the space made by the form.

Solution

Go to the second principle and what you call form is a figure made of a long line and long points, and since a figure is a likeness of form, a long point signifies form more strongly than matter.

Question 3

When a body divides into several parts, does the form of the body have passion before its matter?

Objection

Passion only has to do with matter, and not with form. Therefore, when a body is divided, it has passion in matter but not in form.

Solution

Go to the third principle, which declares that form has passion before matter when a body is divided, because a long point is more divisible than a wide point. Moreover, you are wrong in saying that the form of a divided body has no passion, since form is a part of the body, and the entire body has passion when it is divided. Hence, it follows that all parts have passion. However, you are right in saying that passion does not affect form as such directly, but only accidentally, due to the matter it informs, and because matter has passion when the form leaves the body to decompose into its parts, form also undergoes passion and corruption, as form depleted of its own matter fails.

Question 4

Is an angle made of long points?

Objection

An angle cannot consist of long points, because if it were made of long points, it would make a flat, straight line and the angle could not be what it is.

Solution

Go to the fourth principle, and you are right, supposing that the angle is not made of curved points.

Question 5

Do subtle and slender trees have qualities as tempered as those of short, thick ones?

Objection

The more a line is long and slender and vertical, the less matter it has than a short, thick and transversal line, and therefore the temperament of qualities must be greater in tall, slender trees than in short, thick ones.

Solution

Go to the fifth principle, and you are wrong, because the less that matter resists form, the more intense is its action, like in a choleric man, in whom form has more vigor than in a sanguine man.

Question 6

Why are trees thick at the bottom and slender at the top?

Objection

The reason why trees are thick at the bottom and slender at the top is that the long lines making them up are slender at the top and thick at the bottom.

Solution

Go to the sixth principle, and you are right, but the main reason is in what the corollary of the adduced principle says.

Question 7

Can one part of a body be in another?

Objection

Composition is what enables one part of a body to be in another so that parts mutually contain each other.

Solution

Go to the seventh principle, and you are right with regard to composition and what follows upon the entry of parts into each other. Nevertheless, the main reason has to do with locus, capacity and the influence of qualities and properties that elements have on each other, while these properties do not abandon their proper subjects, as the heat of fire does not abandon fire that is its own subject. Moreover, these properties are habituated and configured with the circular, square and triangular figure.

Question 8

What is a perfectly full body?

Objection

The body of a bottle or some other container completely full of water, wine or some other liquid is a perfectly full body, because a bottle cannot contain more wine than its full capacity allows.

Solution

Go to the eighth principle, and although the bottle, or other container is full of wine, it does not follow that it is a full body due to the wine, given that the wine is external to the body of the bottle. However, what mainly make the body of the bottle full are its own parts, and each of its parts is full of other parts. Moreover, the full part is full like watered down wine is full of water, and conversely, the water is full of wine. The fullness of veins is naturally closer to perfection when they are full of blood as it entirely fills a living body.

Question 9

In generation, does nourishment and growth begin sooner with a long line, than with a wide or deep one?

Objection

It is obvious that in the roots of a tree, nourishment and growth begin sooner in depth than with long or wide lines, given that the tree first extends its roots downward.

Solution

Go to the ninth principle, and you are right inasmuch as a tree begins to extend its roots downward in a long line, and depth is the locus where the long line extends.

Question 10

Is the last point of a line long?

Objection

Since a line terminates between two points, its terminal points, which are of its own essence, must be short points, because if they were long, the line would have to be indeterminate.

Solution

Go to the tenth principle, and you are wrong in saying that a line would have to be indeterminate if the points at its ends were long, given that its points can terminate at a wall whose surface is made of flat and narrow points.

Width

Questions about Width

Question 1

Can width consist of length and depth?

Objection

Length, width and depth are accidents, and as one accident cannot arise from another, width cannot consist of length and depth.

Solution

Go to the first principle of width, and you are right inasmuch as a body abandons one accident and takes on another, as a white body can become black, and the skin of a coat can be widened, and then narrowed and lengthened.

Question 2

Are clouds as thick as they are wide?

Objection

Since clouds are made of ascending vapors, they must be thicker than they are broad or wide, because they make a figure of depth as they ascend.

Solution

Go to the second principle, and you are right in saying that they are thicker than they are wide as they ascend. This is because of the rising vapors, but once the clouds have formed, they are broader than they are thick because the points whereby they are tempered are between ascent and descent, and so they move transversally between upward and downward directions.

Question 3

Which element naturally has the widest points?

Objection

No element has points as wide as water has, and this is obvious because water makes a flatter and more extended surface in the sea, or a pond, than any other element in the same situation.

Solution

Go to the third principle, and you are wrong because the surface of air is on the surface of the sea.

Question 4

Is width more suited to angles than are length and depth?

Objection

An angle as such is deeper than it is long or wide, and this the eye can see in the point terminating the angle.

Solution

Go to the fourth point, and you are right with regard to the final point of an angle, but you are wrong with regard to the angle as a whole.

Question 5

Is width always a part of some figure?

Objection

Some width is not part of a figure, as the world, that can be infinitely wide, but has no shape.

Solution

Go to the fifth principle, and you are wrong, given that width must always have a body as a subject, and all bodies must have a round figure with angles, and no round figure with angles can be without finite quantity.

Question 6

What causes winds?

Objection

Winds are caused by motion moving air from place to place.

Solution

Go to the sixth principle, and you are right, but the question is about the cause of the motion itself, which the adduced principle indicates.

Question 7

Does color first begin with a point, or a line?

Objection

All color on a surface is visible, and all surfaces are wide and made of lines, so that the first beginnings of color must be the constituted lines that make up the width sustaining the color.

Solution

Go to the third principle, and you are right with regard to secondary principles, but not with regard to first principles. Now points are the material principles of lines, as can be seen in an apple when its color begins to alter and decay. It first begins with a round point of altered color, which it would not do if the first principles of color were lines, for then the apple's color would sooner change in one long line across the entire surface, etc.

Question 8

As air contracts, does it lose its width?

Objection

Breadth and contraction are different and contrary; therefore, as air's matter leaves its wide habit and puts on a narrow habit, it must lose its wide habit.

Solution

Go to the eighth principle. Now you would be right if width was a substance. However, as it is an accident, it goes on to another subject that has a narrow habit.

Question 9

In what season does air have the greatest width?

Objection

Fire and water are contraries, and because in winter, water restricts air, fire makes it wider in the summer than at any other time of year.

Solution

Go to the ninth principle, and you are right inasmuch as air is wider in summer than in winter. However, in spring, it is wide by its own nature, and in summer it makes a line that is longer than it is wide, because length is more suited to fire than width, since its sphere is made of long lines more than the sphere of air, and fire is divisive, and has less matter than air.

Question 10

Is width made of points?

Objection

Width cannot consist of points, given that width always has quantity, whereas a point has no quantity.

Solution

Go to the tenth principle. You are right in saying that width always has quantity, but not in negating that points have quantity. Because natural points hold potential quantity, and as one point mixes with another, the quantity of each comes into act. Now the subject of this quantity is the line made of the corporealities of points inasmuch as substance is its subject, and its quantity is made of quantities of points, and as one line joins another, width is brought from potentiality into act, and consequently, the transversal line and the surface of the body that were potential are made actual.

Depth

Questions about Depth

Question 1

Is the center more desirable due to length, or depth?

Objection

We clearly see that in plants, appetite moves the top parts up from the depth below, as when a tree collects in its roots the matter for the fruit that it brings to its destination in the branches through a long line from bottom to top, and so it is obvious that appetite is greater in length than in depth.

Solution

Go to the first principle of depth. Now you are right with regard to the center of the earth, but not with regard to the center of fruit. In the center of a fruit, there is more appetite in depth than in length, given that a tree's appetite seeks above all to conserve its species, which it conserves in the seed in the middle of its fruit, as is the case with apple seeds and other seeds.

Question 2

Does depth arise from a point, or a line?

Objection

Width arises from the conjunction of lines that have a locus tempered by lightness and concavity, and depth arises from the concavity of lines, not from a point, given that points are not subject to concavity.

Solution

Go to the second principle, and you are right about the principles of width, but you are wrong in saying that depth arises not from a point but from a concave line. Now your reason is not valid, given that an acute angle can be more acute than a line, and this acute angle has an acute point at the surface of its tip, and this is an actual acute point, brought into composition by many points whose actual concavity is in the common acute point that causes depth.

Question 3

Is there discrete quantity in any point?

Objection

All points are simple, given that no point is a body, and since all points are simple, and there can be no discrete quantity without composition, there is no discrete quantity in any point.

Solution

Go to the third principle. Now you are wrong in negating that there are compound points, which must exist to make up a long line together with many other points. Hence, width can consist of many collateral long lines, so that depth can consist of length and width, and bodies can be full in all three dimensions.

Question 4

Through what door can one point enter into another?

Objection

The door through which one point can be in another is motion sustained in many points moved into each other by the natural agent moved by natural motion.

Solution

Go to the fourth principle, and you are right with regard to composition and mixture, but depth must be the first door through which one point can be in another, just as in time, many moments can be in the same instant, although the moments occur in different places and substances.

Question 5

What is the first door of generation?

Objection

The first door of generation is passion, given that form brought from potentiality into act first enters into the passion of the subject that potentially held it, from which the agent draws it forth and actualizes it with passion.

Solution

Go to the fifth principle, and you are right with regard to common matter subject to generation. However, the generation of a particular generated thing must begin with single points that form a community that comes from potentiality into act from its points, so that the first door of generation must be depth extended into deep points, and through depth, points can enter into each other, and this entry is the beginning of composition and generation with passion.

Question 6

Is depth an accident or a substance?

Objection

Depth must be a substance, not an accident, for if it were an accident, it could not contain anything corporeal.

Solution

Go to the sixth principle, and you are right in saying that depth could not contain anything corporeal in itself if it were an accident, but depth is nonetheless an accident, as it is an instrument whereby one body can contain another, just as heat is an instrument of fire enabling it to heat other elements.

Question 7

Is there some indivisible point that is the terminus of many angles?

Objection

No point can be indivisible, nor simply be the terminus of many angles, given that in every point there is depth whereby it has angles in itself, and in which and through which one point can enter into another so that they all make up one substance with one line, surface and continuous depth.

Solution

Go to the seventh principle. Now you are right with regard to the continuity of substance made of points and lines. However, you are wrong inasmuch as it constitutes a center standing in the middle as the terminus of angles. Without this terminus, no single point can be the common subject of many lines with their angles, just as in a square figure, if the major diametric lines did not terminate in one common center, they could not take on the shape of angles.

Question 8

Why is depth more disposed to science than are length and width?

Objection

The reason why depth is more disposed to science than are length and width is that the intellect desires to enter into the depths of the true secrets of substance.

Solution

Go to the eighth principle, and you are right with regard to spiritual depth, but this question is about depth as a habit of bodies.

Question 9

Does the sea have a center as deep as the earth's center is?

Objection

In water, points and lines are more continuous than in earth, and its acute points can be longer. Therefore, due to the circumferences of the elements, water must have a center deeper than earth's.

Solution

Go to the ninth principle, and your objection is not valid because water cannot make any angles on its own without earth, which it would make if it had a center deeper than earth's.

Question 10

Does feeling have more to do with depth than with length?

Objection

Length and width are in the extremities of sensible substances, whereas depth is in the middle. Moreover, because matter is more confused at the center than in the extremities, its outer parts are more sensible to taste than its inner parts, and thus, feeling is greater in length than in depth.

Solution

Go to the tenth principle, and you are right with regard to local sensibility, but not with regard to the sensation of closeness, which has more to do with the depth of the sensor and the sensed than with the length and width of the sensor and the sensed.

We have dealt with 100 natural questions applied to this treatise on Geometry with its 100 principles and their corollaries. What remains now is to make questions about the 40 figures, which you can solve by using the things we said about them in the second part of this science.

Epilogue

This New Geometry is now finished, to the glory and praise of Our Lord from whom all graces and good things come.

It is new because it a new discoverey with a new method for investigating corporeal measurements, as shown in its figures, principles and questions. This science is useful, as we said at the beginning of the second book, and it is a science easy to learn, and through it, one can gain better access to other sciences.This science was finished in Paris in the month of July, in the year of Our Lord 1299.

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References

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